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The effective field theory of codimension-two branes

Claudia de Rham
Dept. of Physics & Astronomy, McMaster University,
1280 Main St. W., Hamilton ON, Canada, and
Perimeter Institute for Theoretical Physics,
31 Caroline St. N., Waterloo, ON, Canada
E-mail: crham@perimeterinstitute.ca

Abstract: Distributional sources of matter on codimension-two and higher branes are only well-defined as regularized objects. Nevertheless, intuition from effective field theory suggests that the low-energy physics on such branes should be independent of any high-energy regularization scheme. In this paper, we address this issue in the context of a scalar field model where matter fields (the standard model) living on such a brane interact with bulk fields (gravity). The low-energy effective theory is shown to be consistent and independent of the regularization scheme, provided the brane couplings are renormalized appropriately at the classical level. We perform explicit computations of the classical renormalization group flows at tree and one-loop level, demonstrate that the theory is renormalizable against codimension-two divergences, and extend the analysis to several physical applications such as electrodynamics and brane localized kinetic terms.

Keywords: Field Theories in Higher Dimensions, Large Extra Dimensions, Renormalization Group.

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Contents

1. Motivations

Over the past ten years, large (supersymmetric) extra dimensions have been subject to an increased attention, providing a simple framework for new cosmological ideas. Motivated by scenarios such as the Randall-Sundrum model [1], physics in the presence of one extra dimension has been extensively analyzed and represents an interesting framework in which effects from the higher dimension can be understood. Nevertheless, just as the dynamics of domain walls in a four-dimensional spacetime is in many ways very different to that of a cosmic string or point particle, the behavior of gravity near codimension-one branes is not representative of that around higher codimensional objects. Models with two large extra dimensions, on the other hand, are capable of tracking some of the more interesting features of higher codimension objects, without introducing complications associated with higher-codimension branes. Six-dimensional (super)gravity is therefore a choice framework for the study of higher-dimensional effects, and presents remarkable features of its own. Solutions of six-dimensional gravity, have been found in refs. [2, 3], cosmological solutions in [4], and the stability of these models has been studied in [5, 6]. In six-dimensional supergravity, for instance, not only would the Hierarchy problem be resolved if these dimensions had a submillimeter size [7], but if supersymmetry remained unbroken in the bulk at energies much lower that on the brane, the Casimir energy could also be of the same order of magnitude as the observed four-dimensional cosmological constant, [8]. Another property of codimension-two objects relevant to the cosmological constant problem is their capacity of preserving a flat Minkowski induced geometry in the presence of any tension, [9 – 12].

Codimension-two branes in the context of six-dimensional (super)gravity can therefore provide potential resolutions of two of the most embarrassing problems of current particle physics and cosmology, namely the Hierarchy and cosmological constant problems in scenarios where the Weinberg's argument has different imports, [15]. Nevertheless, these models are faced with one great obstacle, the necessity of regularizing the brane before any question can be addressed, [16]. Distributional sources of matter on codimension-two and higher branes are indeed only well-defined as regularized objects, and one can therefore wonder whether any regularization-independent statement can even be made. From a field theory perspective, one expects the low-energy theory on such a brane to be independent of any high-energy regularization scheme, yet as of today, no regularization-invariant scheme has been proposed to study the effective theory on such branes. The only known work in this direction has been developed by Goldberger and Wise in 2001 (see ref. [17]), where it is pointed out that a field living on a six-dimensional flat spacetime will typically present a pathological behavior if coupling terms were to be introduced on a codimension-two surface. This pathology can however be removed by appropriate renormalization of the coupling constants. In this paper, we propose a direct extension to ref. [17], where we analyze couplings between bulk fields, free to live in the entire six dimensions, and brane fields, which are confined to codimension-two branes. The couplings between the two fields induce pathologies for both fields which can be absorbed by appropriate renormalization. Similar ideas have been proposed as being useful for understanding black hole physics [18], post-newtonian corrections [19] and brane localized kinetic terms [20].

In what follows, we first review in section 2 the problems arising when dealing with distributional sources on codimension-two branes, the regularization schemes that have been proposed in the literature as well as different sources of confusion which we clarify. We then explain the philosophy of our approach and discuss the main consequences. Our strategy is applied in section 3, where a scalar field toy-model is considered. In particular, we analyze couplings between a bulk and a brane scalar field and discuss the renormalization procedure using two different techniques. The first one makes use of a conical cap to regularize the brane, while the other removes the divergences directly in the propagators. Both methods give rise to the same Renormalization Group (RG) flows. This analysis is then extended to all possible relevant and marginal couplings in section 4, where the three and four-point functions are computed as well as the loop diagrams. Using these couplings,

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we also demonstrate that the theory is renormalizable. The second part of this paper is then dedicated to the physical implications. In section 5, we show how the same prescription remains valid when considering the more physical example of interactions between gravity and electromagnetism and finally explores the implications for localized kinetic terms on the brane which are relevant for models such as the Dvali-Gabadadze-Porrati (DGP) in section 6. We show how to make sense of kinetic couplings on the brane and deduce that only fixed functions of the kinetic terms are allowed by the renormalization procedure. After concluding in section 7, we consider all possible local counterterms in appendix A and argue that no such counterterms can simultaneously absorb the bulk field divergences both in the bulk and on the brane, in complete agreement with our procedure. Using the RG flows obtained for the brane couplings, we finish in appendix B by computing the arbitrary N-point function at all order in loops, and prove that it remains finite in the thin-brane limit, hence justifying that the theory is renormalizable against codimension-two divergences.

2. Understanding gravity on codimension-two branes

2.1 Distributional sources

In 1987, Geroch & Traschen showed that strings and point particles do not belong to the class of metrics whose curvatures are well defined as distributions, [16]. In their analysis, they considered a string in $(3+1)$-dimensions to be regularized as a cylinder of radius ϵ carrying energy density ρ. The profile of the gravitational potential U is obtained by solving Poisson's equation $\nabla^2 U = -\rho$, where ρ vanishes outside the cylinder. Although well-defined when the cylinder has a finite width ϵ, it turns out that the gravitational potential U is not locally integrable in the thin-brane limit $\epsilon \to 0$ and so strings (and point particles) are not permitted as sources in $(3+1)$-dimensions, unless they are pure tension strings. Issues arising from smoothing out codimension-two branes are also discussed in [21].

Despite the amount of attention branes have recently received, mainly motivated by string theory, the situation is unfortunately no different for those objects whose intrinsic codimension is equal or greater to two. From the string theory point of view, progress in these areas has mainly been achieved by neglecting the backreaction of such objects, treating them as test particles, the so-called probe-brane approximation. From a cosmological point view, however, such a procedure would miss some of their most important features and is hence not always satisfying. Instead, much effort has been invested in specific regularizations of the theory, such as models arising from Abelian-Higgs theory, [22], thick brane regularizations, [23], capped branes, [6, 24], intersecting branes, [25], codimension-two branes confined on codimension-one objects [26], etc...

In all of these examples, if the extra dimensions are compact or if an Einstein-Hilbert term is confined on the brane, one recovers four-dimensional gravity for the zero mode. However a logarithmic divergence appears in the first Kaluza-Klein mode as soon as the regularization mechanism is removed (or the thin-brane limit is taken). Understanding the significance of this divergence and the consequences for an observer on the brane represents the main objective of this paper.

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2.2 Philosophy

Bulk fields away from a defect should be insensitive to the regularization procedure, but evaluating them on the defect itself requires knowledge of the internal structure of the defect. The philosophy of this paper is therefore to accept the presence of divergences in the thin-brane limit for bulk fields when evaluated on a codimension-two defect but to ensure that these divergences do not propagate into matter fields confined on the defect. We will show the validity of an effective field theory for such fields and present how observables on the brane remain finite after appropriate renormalization of the coupling constants.

Origin of the problem. Following the analysis of Goldberger and Wise [17], the brane-to-brane Feynman propagator for a massless scalar field living in six-dimensional flat space-time with a conical singularity of deficit angle $2\pi(1-\alpha)$ at $r=0$ is given by

$$D_k(0;0) = -\int_0^\Lambda \frac{\mathrm{d}q q}{2\pi\alpha} \frac{i}{k^2+q^2} = -\frac{i}{2\pi\alpha}\log\frac{\Lambda}{k}\,, \tag{2.1}$$

where k is the four-dimensional momentum along the brane direction, (see eq. (3.18) for more details.) The brane-brane propagator is therefore divergent in the thin-brane limit for which the cutoff Λ is sent to infinity (or for large physical scale.) In real space, on the other hand, the free propagator is finite outside the coincidence limit, and the presence of the logarithmic divergence in four-dimensional momentum space is merely a consequence to the fact that the gravitational potential of six-dimensional gravity behaves as x^{-3} in real space. More precisely, one can express the free brane-to-brane bulk propagator in real space as

$$\begin{aligned} D(0,x;0,x') &= \int \frac{\mathrm{d}^3k}{(2\pi)^3}\int_0^\infty \frac{\mathrm{d}q\, q}{2\pi\alpha}\frac{e^{i\mathbf{k}\,.\,(\mathbf{x}-\mathbf{x}')}}{k^2+q^2} \\ &= \int_0^\infty \frac{\mathrm{d}q\, q}{2\pi\alpha}\frac{e^{-q\,|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|} = \frac{1}{8\pi^2\alpha}\frac{1}{|\mathbf{x}-\mathbf{x}'|^3}\,, \end{aligned} \tag{2.2}$$

where x and x' represent directions tangent to the codimension-two brane and the two-point function is evaluated at $r=0$ along the normal direction. When the integral over the brane momentum k is performed before that over the bulk momentum q, the two-point function is finite everywhere. Nevertheless, as soon as a source is considered at $r=0$, the logarithmic behavior of the brane-brane two-point function in momentum space becomes relevant. This will be seen more concretely in what follows.

General sources of confusion. We present here two general sources of confusion that usually appear when discovering these logarithmic divergences:

- The first one is related to the nature of the divergence, and the order of magnitude at which it arises. Any codimension-two object arises from an underlying theory (*e.g.* Abelian-Higgs field theory, string theory or any other underlying theory) which will naturally provide a regularization mechanism for the brane. We could hence argue that the notion of thin-brane limit is not physical and one should not be concerned about any thin-brane divergences. Yet, such a argument would be going against the principles of Effective Field Theory (EFT). Even though the brane is expected to be regularized at some scale (*e.g.* the string scale), we expect from EFT that the low-energy physics is independent of the high-energy regularization mechanism. In other words, one should not need to understand the physics at string scale in order to understand and make predictions about the low-energy physics.

- Faced with this realization, one can hope that introducing brane localized counterterms should remove the logarithmic divergence present in (2.1) without effecting the bulk propagator. Unfortunately such an approach is too naive in this context, since brane-localized counterterms will not only affect the brane-brane propagator $D_k(0,0)$ but also the brane-bulk $D_k(r,0)$ and bulk-bulk propagators $D_k(r,r')$ which were previously finite. Any attempt to absorb the logarithmic divergence of the brane-brane propagator into brane localized counterterms will then automatically result in the introduction of further divergences. This argument is made more explicit in appendix A, where we consider the most general set of local counterterms (that remain quadratic in the scalar field) both in the bulk and on the brane, and show explicitly that no such counterterms will allow the propagator to be finite everywhere. In the philosophy we will follow, we will thus not attempt to make the bulk field propagator finite everywhere but will rather explore the consequences for observers on the brane.

Strategy. In a conical space-time, the propagator diverges at the tip of the cone. The aim of this paper is to explore the consequences for a scalar field living on the tip and coupled to a bulk field. We expect physically that

1. Bulk fields evaluated away from the brane should not depend on the regularization mechanism and thus be finite in the thin-brane limit.

2. Bulk fields evaluated on the brane itself are sensitive to the regularization procedure, since the position at which they are evaluated, *i.e.* the position of the brane, is dependent of the regularization. Therefore we do not require bulk fields evaluated on the brane (at $r = 0$) to be finite in the thin-brane limit.

3. Brane fields, should have a well defined low-energy theory independent of the brane regularization. As long as the energy scales probed by an observer on the brane are much lower than that of the cutoff theory, the physics we will observe should be independent of the regularization, and hence finite in the thin-brane limit.

In this paper, we will follow this philosophy carefully. The case of bulk fields with brane couplings was considered in [17]. We here extend this analysis to matter fields confined to the brane and draw conclusions for brane observers.

3. Scalar field toy-model

In this section we compute the propagator for scalar fields confined to a codimension-two brane and coupled with a bulk scalar field.

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We work in a six-dimensional flat space-time with a conical singularity located at $r = 0$:

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}r^2 + r^2\mathrm{d}\theta^2, \tag{3.1}$$

with $0 \leq \theta < 2\pi\alpha$, and where $2\pi(1-\alpha)$ is the deficit angle ($\alpha \leq 1$). A three-dimensional brane is located at the tip of the cone and x^μ represents the coordinates along the brane direction. We use the notation $a = 0, \ldots, 5$; $\mu = 0, \ldots, 3$;
$y = (\theta, r)$ and $x^a = (x^\mu, y)$.

This scalar field toy-model represents a good framework for the study of effective theories on codimension-two branes. The theory is composed of two coupled scalar fields, namely

- The scalar field ϕ which symbolizes the bulk fields (gravity, dilaton, gauge field...) and thus lives in six dimensions,
- The brane field χ which symbolizes the matter fields living on the brane (standard model) and thus confined to a four-dimensional space-time.

The action for this system can thus be taken to be of the form

$$S = -\int \mathrm{d}^6 x \left(\frac{1}{2}(\partial_a\phi)^2 + \delta^2(y)\left[\frac{1}{2}(\partial_\mu\chi)^2 + \frac{m^2}{2}\chi^2 + \lambda_2\phi^2 + \lambda\chi\phi\right]\right), \tag{3.2}$$

where for simplicity we have assumed the field ϕ to be massless in the bulk, but further extensions will be considered in section 4. The coupling between the bulk and brane fields is symbolized by the term $\lambda\chi\phi$ (which can be set to zero). Higher interactions will be considered in section 4.

To make contact with previous works in the literature, we first consider a specific thick-brane regularization mechanism, and show how the coupling constants can be renormalized in order for the theory to remain finite in the thin-brane limit. We then explore the renormalization mechanism in a more systematic way by analyzing the different classical two-point functions before turning to interactions and one-loop corrections in the following section. We point out that both methods will give rise to the same tree-level renomalized couplings.

3.1 Thick-brane regularization

As a warm up, we follow a standard technique used in the literature to confine matter fields on a codimension-two brane, namely a thick-brane regularization in which the brane is no longer located at $r = 0$, but rather at $r = \epsilon$. The thin-brane limit is then recovered when $\epsilon \to 0$. The action (3.2) is regularized by

$$\begin{aligned} S = -\int \mathrm{d}^4 x \mathrm{d}\theta \mathrm{d}r\, r \Bigg(& \frac{1}{2}(\partial_r\phi)^2 + \frac{1}{2}(\partial_\mu\phi)^2 \\ & + \frac{\delta(r-\epsilon)}{2r\pi\alpha}\left[\frac{1}{2}(\partial_\mu\chi)^2 + \frac{m^2}{2}\chi^2 + \lambda_2\phi^2 + \lambda\chi\phi\right]\Bigg), \end{aligned} \tag{3.3}$$

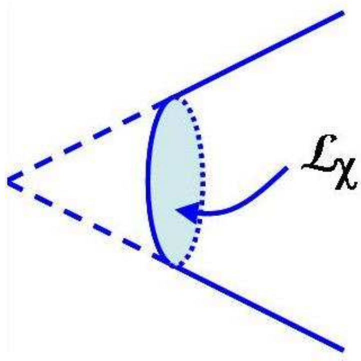

Figure 1: Thick brane regularization

where for simplicity we omit for now any angular dependance. This leads to the following equations of motions:

$$-\frac{1}{r}\partial_r(r\phi'(r)) + k^2\phi = -\frac{1}{2\pi\alpha r}\left(\lambda\chi + \lambda_2\phi\right)\delta(r-\epsilon) \tag{3.4}$$

$$\delta(r-\epsilon)\left[(k^2+m^2)\chi = -\lambda\phi\right], \tag{3.5}$$

k^2 being the eigenvalue of the four-dimensional d'Alembertian $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$.

Integrating the first equation along the brane gives rise to the following jump condition:

$$r\phi'(r)\big|_{\epsilon^-}^{\epsilon^+} = -\frac{1}{2\pi\alpha}\left(\lambda\chi + \lambda_2\phi\right). \tag{3.6}$$

One can solve for the bulk scalar field separately in the conical cap (for $0 < r < \epsilon$) and within the bulk ($r > \epsilon$). We choose the solution within the cap such that the scalar field remains finite at the tip $r = 0$. This leads to the following solutions

$$\phi(r) = \begin{cases} AI_0(kr) & \text{for } r < \epsilon \\ I_0(kr) + BK_0(kr) & \text{for } r > \epsilon \end{cases}, \tag{3.7}$$

where I and K are the two modified Bessel functions, or hyperbolic Bessel functions, and I_0 remains finite as $r \to 0$. For $r < \epsilon$, we have set the coefficient of the divergent Bessel function K to zero, so that $\phi(r)$ remains finite as $r \to 0$. For $r > \epsilon$, on the other hand, no such choice has been made and this solution is therefore independent of any other boundary conditions. These results will thus stand independently to any condition imposed on the fields away from the brane.

The constants A and B are determined using the boundary condition (3.5) and the jump condition (3.6). In the thin-brane limit, this leads to

$$A = \frac{2\pi\alpha(k^2+m^2)}{2\pi\alpha(k^2+m^2) - (\lambda^2 - \lambda_2(k^2+m^2))(\Gamma + \log\frac{k\epsilon}{2})}, \tag{3.8}$$

$$B = \frac{-\lambda^2 + \lambda_2(k^2+m^2)}{2\pi\alpha(k^2+m^2) - (\lambda^2 - \lambda_2(k^2+m^2))(\Gamma + \log\frac{k\epsilon}{2})}, \tag{3.9}$$

where Γ is the Euler number, $\Gamma \simeq 0.57$.

For the bulk scalar field to be well-defined in the thin-brane limit, we require B to remain finite when probing large physical scales $k\epsilon \to 0$. This will only be possible if the logarithmic divergence is reabsorbed into one of the coupling constants λ, λ_2 or m. Furthermore the scalar field χ should also be well-defined in that limit. This will thus be the case if both the following quantities remain finite

$$B/\chi = \lambda - \frac{\lambda_2}{\lambda}(k^2 + m^2) \tag{3.10}$$

$$\chi^{-1} = -\frac{1}{\lambda}(k^2 + m^2) + \frac{1}{2\lambda\pi\alpha}\left(\lambda^2 - \lambda_2(k^2 + m^2)\right)\left(\Gamma + \log\frac{k\epsilon}{2}\right). \tag{3.11}$$

The logarithmic divergence can thus be absorbed into the coupling constants by arguing that their renormalized expression is related to their bare value by

$$\lambda_2 = \frac{\lambda_{2b}}{1 - \frac{\lambda_{2b}}{2\pi\alpha}\log\rho\epsilon}, \qquad \lambda = \frac{\lambda_b}{1 - \frac{\lambda_{2b}}{2\pi\alpha}\log\rho\epsilon} \quad \text{and} \quad m^2 = m_b^2 + \frac{\lambda^2}{\lambda_2}, \tag{3.12}$$

where the subscript b represents the bare value, and ρ is the physical scale. We therefore get the following renormalization group flows for the brane couplings

$$\rho\frac{\mathrm{d}\lambda_2}{\mathrm{d}\rho} = \frac{\lambda_2^2}{2\pi\alpha}, \qquad \rho\frac{\mathrm{d}\lambda}{\mathrm{d}\rho} = \frac{\lambda\lambda_2}{2\pi\alpha} \quad \text{and} \quad \rho\frac{\mathrm{d}m^2}{\mathrm{d}\rho} = \frac{\lambda^2}{2\pi\alpha}. \tag{3.13}$$

Notice that we recover the same flow as ref. [17] for the coupling λ_2 which gives rise to a mass term for the bulk field on the brane. The renormalization of this coupling ensures that the bulk field to be finite away from the tip. However we wish to emphasize that this procedure does not get rid of the divergence of ϕ as $r \to 0$. The point of the renormalization is to make sense of the bulk field away from the tip, however at the tip itself the bulk field diverges logarithmically as explained in section 2 which is consistent with our philosophy. The key point here is that one can still make sense of the brane field χ (χ is finite) despite its coupling with ϕ. In other words, at low-energy, matter fields living on a codimension-two brane are independent of the regularization procedure, even though they couple to gravity and other bulk fields which are themselves ill-defined in the brane in the thin-brane limit. This is possible through adequate renormalization of the couplings.

The renormalization for the coupling λ_2 is already known from ref. [17]. We show here how the renormalization is extended to the couplings for brane fields. In particular, we see that as soon as $\lambda \neq 0$, the brane field acquires a mass.

In this setup, we have used an artificial thick brane regularization. More fundamentally, we expect this defect to arise as the result of other fields (*e.g.* Abelian-Higgs scalar and gauge field Φ, A_μ), providing a natural regularization. The fundamental theory is thus of the form

$$\mathcal{P} = \int \mathcal{D}[\Phi]\mathcal{D}[A_\mu]\mathcal{D}[\phi]\mathcal{D}[\chi]e^{iS_{\rm tot}[\Phi, A_\mu, \phi, \chi]}. \tag{3.14}$$

The resulting field theory (3.2) is obtained by integrating out the regularizing fields Φ and A_μ. In this picture, we therefore expect that field loop integrations generate the same tree-level counterterms as those obtained in (3.12).

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In what follows, we shall recover the same results using a Green's function approach, this uses the same technique as in [17]. We will also discuss other interaction terms and show how the same renormalization procedures goes through, leading to a renormalizable theory.

3.2 EFT approach and tree-level renormalization

In this section we adopt a more field theoretic approach and require that the bulk-bulk propagator of the bulk field remains finite as well as the brane field propagator. We will proceed in three steps:

i) We consider first of all the purely free theory for which the bulk and brane fields do not couple and all couplings vanish $\lambda_2 = \lambda = 0$. In particular we recover the logarithmic divergence of the bulk field propagator when evaluated on the brane, but this divergence is only present in four-dimensional momentum space.

ii) We then consider the corrections to the bulk field propagators arising from the mass term λ_2 on the brane. This situation is precisely that considered in [17], and we will follow the same approach. In particular, we will show how this brane coupling induces divergences in the bulk which can be removed by appropriate renormalization of the coupling λ_2, hence recovering the same result as in (3.12).

iii) We finally consider the corrections to both the bulk and brane field propagators induced by the coupling λ between the two fields. Once again, these couplings will induce divergences which can be removed by renormalization of λ and m^2, as in (3.12).

i) *Free propagators* We concentrate first of all on the purely free theory given by the action

$$S = -\int \mathrm{d}^4x\mathrm{d}\theta\mathrm{d}r\ r\left(\frac{1}{2}(\partial_a\phi)^2 + \delta^2(y)\left[\frac{1}{2}(\partial_\mu\chi)^2 + \frac{m^2}{2}\chi^2\right]\right). \tag{3.15}$$

The propagators for both fields satisfy

$$r\Box_x^{(6d)}D(x^a,x'^a) = \left[\partial_r(r\partial_r) + \frac{1}{r}\partial_\theta^2 + r\Box_x\right]D(x^a,x'^a) = i\delta^{(6)}(x^a - x'^a) \tag{3.16}$$

$$\Box_x H(x^\mu,x'^\mu) = i\delta^{(4)}(x^\mu - x'^\mu)\,, \tag{3.17}$$

where D is the Feynman propagator for ϕ and H the one for χ. Using a mixed-representation, *i.e.* momentum space along the directions x^μ and real space along the two extra dimensions, the propagators for both fields are simply

$$D_k(r,\theta;r',\theta') = -\sum_{n=-\infty}^{+\infty}\int_0^\infty \frac{\mathrm{d}qq}{2\pi\alpha}\frac{i}{k^2+q^2}e^{i\tilde{n}(\theta-\theta')}J_{|\tilde{n}|}(qr)J_{|\tilde{n}|}(qr') \tag{3.18}$$

$$H_k = -\frac{i}{k^2+m^2}\,, \tag{3.19}$$

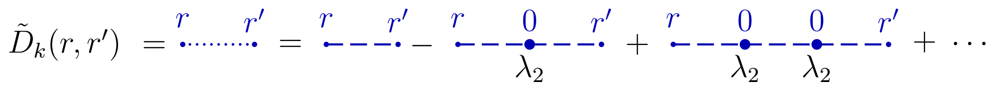

Figure 2: Corrections to the bulk field two-point function arising from the brane mass term λ_2. The blue dashed lines represent the free bulk field propagator $D_k(r, r')$ while the dotted line is that corrected for the mass term *i.e.* $\tilde{D}_k(r, r')$.

where J_n the Bessel function of first kind, $\tilde{n} = n/\alpha$, and $k^2 = \eta^{\mu\nu} k_\mu k_\nu$ the four-dimensional momentum.

Notice that in this representation, *i.e.* in four-dimensional momentum space, the free propagator for ϕ is finite when at least one of the points is evaluated in the bulk (*i.e.* $D_k(r, r')$ and $D_k(r, 0)$ finite) but it contains a logarithmic singularity when trying to evaluate both points on the brane. Introducing a momentum cutoff scale Λ in the evaluation of the propagator, one has

$$D_k(0,0) = -\int_0^\Lambda \frac{\mathrm{d}qq}{2\pi\alpha} \frac{i}{k^2 + q^2} = \frac{-i}{2\pi\alpha} \log \frac{\Lambda}{k} \,, \tag{3.20}$$

which has the short distance singularity pointed out in [17]. This divergence is usually not a problem since the two-point function is actually finite in real space, (see section 2). However, as soon as a source is included at $r = 0$, the convolution of this two-point function will not be finite in real space. We therefore expect this divergence to affect the two-point function of both scalar fields when brane couplings are included.

ii) *Corrections from the brane mass term* λ_2 The previous two-point functions were that of the free theory for which the both fields were not coupled. We can now "dress" these propagators with first of all the coupling λ_2:

$$S = -\int \mathrm{d}^4x \mathrm{d}\theta \mathrm{d}r \; r \left(\frac{1}{2}(\partial_a\phi)^2 + \delta^2(y) \left[\frac{1}{2}(\partial_\mu\chi)^2 + \frac{m^2}{2}\chi^2 + \frac{1}{2}\lambda_2\phi^2 \right] \right) . \tag{3.21}$$

The propagator for the brane field χ remains unaffected while that for the bulk field ϕ gets modified to

$$\begin{aligned}\tilde{D}_k(r,r') &= D_k(r,r') - i\lambda_2 D_k(r,0) D_k(0,r') + i^2\lambda_2^2 D_k(0,0) D_k(r,0) D_k(0,r') + \cdots \\ &= D_k(r,r') - \frac{i\lambda_2}{1 + i\lambda_2\, D_k(0,0)} D_k(r,0) D_k(0,r') \,, \end{aligned} \tag{3.22}$$

as symbolized in figure 2. If $D_k(0, 0)$ was finite, this bulk field propagator would be finite at tree level as one should expect from usual field theory. In the presented case, the logarithmic divergence of $D_k(0, 0)$ needs to be absorbed in the coupling constant λ_2 in the following way:

$$\lambda_2(\mu) = \frac{\lambda_2(\Lambda)}{1 + \frac{\lambda_2(\Lambda)}{2\pi\alpha} \log \frac{\Lambda}{\mu}} \,, \tag{3.23}$$

so that this coupling constant flows as

$$\mu \frac{\mathrm{d}\lambda_2(\mu)}{\mathrm{d}\mu} = \frac{\lambda_2^2(\mu)}{2\pi\alpha} \,. \tag{3.24}$$

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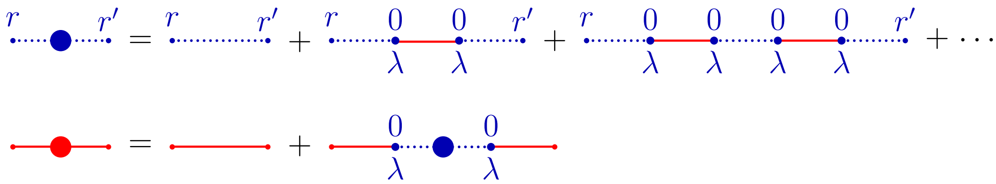

Figure 3: Coupling corrections to the two-point functions. The blue dotted lines represent the propagator for the bulk field $\tilde{D}_k(r,r')$, while the red plain lines are the propagator for the brane field χ: H_k. Lines carrying a circle represent the "dressed" propagators $G^{\phi\phi}(r,r')$ (top diagram) and $G^{\chi\chi}$ (bottom) and take into account the tree-level corrections arising from the coupling λ between the bulk and the brane field.

We point out that this renormalization ensures the two-point function $\tilde{D}_k(r,r')$ to be finite away from the brane. However, both the bulk-brane and the brane-brane two point functions remain ill-defined: Both $\tilde{D}_k(r,0)$ and $\tilde{D}_k(0,0)$ contains a logarithmic dependence. Once again, this is to be expected since evaluating the bulk two-point function on the brane requires knowledge about the exact brane position (see section 2).

iii) *Corrections from the coupling between the two fields* Finally, we consider the corrections to these propagators arising from the coupling λ between the bulk and brane fields:

$$S = -\int \mathrm{d}^4x\mathrm{d}\theta\mathrm{d}r\ r\left(\frac{1}{2}(\partial_a\phi)^2+\delta^2(y)\left[\frac{1}{2}(\partial_\mu\chi)^2+\frac{m^2}{2}\chi^2+\frac{1}{2}\lambda_2\phi^2+\lambda\phi\chi\right]\right). \tag{3.25}$$

The tree level Green's functions for this coupled theory are symbolically represented in figure 3.

By summing these diagrams, we obtain the following tree-level Green's functions

$$\begin{aligned} G_k^{\phi\phi}(r,r') &= \tilde{D}_k(r,r') - \lambda^2\tilde{D}_k(r,0)\tilde{D}_k(0,r')H_k\sum_{n\geq 0}(-\lambda^2)^n\tilde{D}_k(0,0)^nH_k^n \\ &= \tilde{D}_k(r,r') - \frac{\lambda^2H_k}{1+\lambda^2H_k\tilde{D}_k(0,0)}\tilde{D}_k(r,0)\tilde{D}_k(0,r') \\ &= D_k(r,r') - \frac{i\lambda_2+\lambda^2H_k}{1+(i\lambda_2+\lambda^2H_k)D_k(0,0)}D_k(r,0)D_k(0,r') \end{aligned} \tag{3.26}$$

$$\begin{aligned} G_k^{\chi\chi} &= H_k\left(1-\lambda^2G_k^{\phi\phi}(0,0)H_k\right) = \frac{H_k}{1+\lambda^2H_k\tilde{D}_k(0,0)} \\ &= \frac{H_k(1+i\lambda_2D_k(0,0))}{1+(i\lambda_2+\lambda^2H_k)D_k(0,0)}\,. \end{aligned} \tag{3.27}$$

Notice that there is now also a mixed two-point function for the bulk and brane fields:

$$G_k^{\phi\chi}(r) = \langle\phi(r)\,,\,\chi\rangle = -\frac{i\lambda H_kD_k(r,0)}{1+(i\lambda_2+\lambda^2H_k)\,D_k(0,0)}\,. \tag{3.28}$$

Here again, if $D_k(0,0)$ was finite, both Green's functions would be finite at the tree level as one expects in usual field theory. In the presented case, the logarithmic divergence of $D_k(0,0)$ needs to be absorbed in the coupling constants. These propagators will thus remain finite in the thin brane limit (for $r, r' > 0$), provided the coupling constants are renormalized as follows:

$$\lambda_2(\mu) = \frac{\lambda_2(\Lambda)}{1 + \frac{\lambda_2(\Lambda)}{2\pi\alpha} \log \frac{\Lambda}{\mu}}, \qquad \lambda(\mu) = \frac{\lambda(\Lambda)}{1 + \frac{\lambda_2(\Lambda)}{2\pi\alpha} \log \frac{\Lambda}{\mu}}, \tag{3.29}$$

$$m^2(\mu) = m^2(\Lambda) - \frac{\lambda^2(\Lambda) \log \frac{\Lambda}{\mu}}{2\pi\alpha + \lambda_2(\Lambda) \log \frac{\Lambda}{\mu}}, \tag{3.30}$$

leading to the following RG flows

$$\mu \frac{\mathrm{d}\lambda_2(\mu)}{\mathrm{d}\mu} = \frac{\lambda_2^2(\mu)}{2\pi\alpha}, \quad \mu \frac{\mathrm{d}\lambda(\mu)}{\mathrm{d}\mu} = \frac{\lambda_2(\mu)\lambda(\mu)}{2\pi\alpha} \quad \text{and} \quad \mu \frac{\mathrm{d}m^2(\mu)}{\mathrm{d}\mu} = \frac{\lambda^2(\mu)}{2\pi\alpha}. \tag{3.31}$$

We therefore recover precisely the same relations between the bare and renormalized coupling constants as in the thick brane analysis (3.12) and the same RG flows (3.13). This is a non-trivial check of our prescription.

Notice that $G_k^{\phi\phi}(r, 0)$, $G_k^{\phi\phi}(0, 0)$ and $G_k^{\phi\chi}(0)$ are still divergent in the four-momentum representation, but the two-point function of the brane field $G_k^{\chi\chi}$ has been made completely finite, and so are $G_k^{\phi\phi}(r, r')$ and $G_k^{\phi\chi}(r)$, for $r, r' \neq 0$.

In summary, we find that by renormalizing the tree-level theory, the propagators of the field on the branes are finite, and the propagator in the bulk are only divergent when one point is evaluated on the brane (and in the coincident limit). Thus there is a consistent effective field theory on the brane and matter can be considered on a codimension-two brane in a completely meaningful regularization-invariant way. This will have important implications for observers on such a brane. Before attacking this argument, let us consider in what follows all possible relevant and marginal interactions between a bulk and a brane field.

4. Renormalization of the relevant and marginal operators

We consider here an extension of the precedent toy-model where further couplings are taken into account. The effective field theory approach will remain completely consistent after the appropriate tree-level renormalization of the couplings. We also expect UV divergences to be present in loop corrections, but these can be dealt with through the usual UV renormalization mechanism.

In order to avoid issues related to the UV divergences, (which are independent of the fact that we consider a codimension-two brane), we restrict ourselves to the relevant and marginal operators. The most general brane interactions are then

$$S = -\int \mathrm{d}^6 x \left[\frac{1}{2} (\partial\phi)^2 + \frac{\delta(r)}{2r\pi\alpha} \left(\frac{1}{2} (\partial\chi)^2 + \frac{m^2}{2}\chi^2 + \lambda\chi\phi + \frac{\lambda_2}{2}\phi^2 + \mathcal{H}_{\chi\phi}^{\mathrm{int}} \right) \right], \tag{4.1}$$

with

$$\mathcal{H}_{\chi\phi}^{\mathrm{int}} = \beta_3\chi^3 + \beta_4\chi^4 + \lambda_3\phi\chi^2, \tag{4.2}$$

where the coupling β_3 is relevant while β_4 and λ_3 are marginal.

For each diagram in this theory, we expect two sorts of divergences to arise:

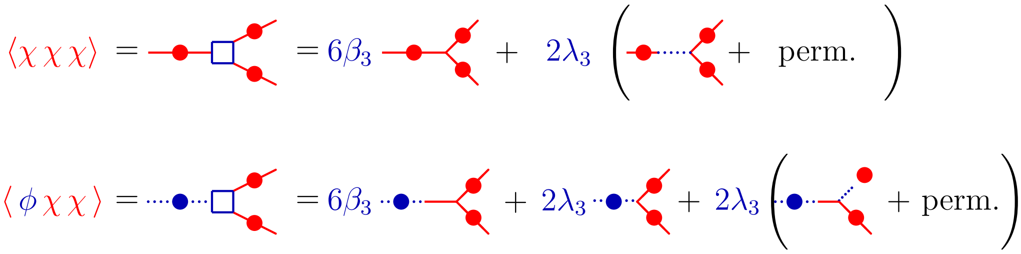

Figure 4: Three-point functions. The blue dotted lines represent the propagator for the bulk field ϕ, while the red plane lines are the propagator for the brane field χ.

- The ones associated with the usual UV divergences which appear in four dimensions when integrating over loops,
- The short-distance divergences associated with the thin-brane limit.

From standard four-dimensional EFT, it is a well-known fact that the interactions of the type $\beta_4\chi^4$ will induce UV divergences in the one-loop correction of both the two-point function and the four-point functions. These divergences can be absorbed by renormalization of the mass m^2, the coupling β_4 as well as the wave-function. However, such divergences can be treated in a completely independent way to that arising at the tree-level in our codimension-two scenario. Interactions of the form $\lambda_3\phi\chi^2$, for instance will typically induce divergences in the thin-brane limit which can be absorbed by appropriate renormalization of the coupling $\beta_3\chi^3$, this will be studied in the three-point functions in what follows.

4.1 Three-point functions

The diagrams involved in the three-point functions are summarized in figure 4. Summing these diagrams, we get, for $\sum_{i=1}^{3}\mathbf{k}_i = 0$,

$$\begin{aligned} G^{\chi\chi\chi}_{k_1,k_2,k_3} &= \langle \chi_{k_1}\chi_{k_2}\chi_{k_3}\rangle \\ &= (-i)\left(6\beta_3 G^{\chi\chi}_{k_1}G^{\chi\chi}_{k_2}G^{\chi\chi}_{k_3} + 2\sum_{i=1}^{3}\lambda_3\, G^{\phi\chi}_{k_i}(0)G^{\chi\chi}_{k_{i+1}}G^{\chi\chi}_{k_{i+2}}\right) \end{aligned} \tag{4.3}$$

$$\begin{aligned} G^{\phi\chi\chi}_{k_1,k_2,k_3}(r) &= \langle \phi_{k_1}(r)\chi_{k_2}\chi_{k_3}\rangle \\ &= (-i)\Big(6\beta_3 G^{\phi\chi}_{k_1}(r)G^{\chi\chi}_{k_2}G^{\chi\chi}_{k_3} + 2\lambda_3 G^{\phi\phi}_{k_1}(r,0)G^{\chi\chi}_{k_2}G^{\chi\chi}_{k_3} \\ &\qquad + 2\lambda_3 G^{\phi\chi}_{k_1}(r)\left(G^{\phi\chi}_{k_2}(0)G^{\chi\chi}_{k_3} + G^{\phi\chi}_{k_3}(0)G^{\chi\chi}_{k_2}\right)\Big), \end{aligned} \tag{4.4}$$

where the factor $(-i)$ arises from the first order expansion of $e^{-i\int \mathrm{d}^4x\,\mathcal{H}^{\rm int}_{\chi\phi}}$. Notice that after appropriate renormalization of λ, m and λ_2, the propagators $G^{\chi\chi}_k$ and $G^{\phi\chi}_k(r)$ have been made finite, however the bulk quantities evaluated on the brane $G^{\phi\chi}_k(0)$ and $G^{\phi\phi}_k(r,0)$ are a priori ill-defined. Once again, this divergence would propagate into the three-point function

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for the brane field, had we not renormalized the couplings β_3 and λ_3. Upon simplification of the previous expression, we find that the divergent part of $\langle\chi\chi\chi\rangle$ is proportional to

$$\langle\chi_{k_1}\chi_{k_2}\chi_{k_3}\rangle_{\rm div} \propto \left(3\beta_3 - \lambda_3\lambda\sum_{i=1}^{3}\frac{iD_{k_i}(0,0)}{1+i\lambda_2 D_{k_i}(0,0)}\right),$$

so that the coupling β_3 should be renormalized as

$$\beta_3(\Lambda) - \frac{i\lambda_3(\Lambda)\lambda(\Lambda)}{1+i\lambda_2(\Lambda)D_k(0,0)}D_k(0,0)$$
$$= \beta_3(\mu) - \frac{\lambda_3(\mu)\lambda(\mu)}{2\pi\alpha + \lambda_2(\mu)\log\frac{\mu}{k}}\log\frac{\mu}{k}\,, \tag{4.5}$$

for any k, and where we recall that the coupling λ has been renormalized in such a way that $\frac{\lambda(\Lambda)}{1+i\lambda_2(\Lambda)D_k(0,0)}$ is finite (see eqs. (3.29), (3.30).) The divergent part of $\langle\phi(r)\chi\chi\rangle$ is then proportional to

$$\langle\phi_{k_1}(r)\chi_{k_2}\chi_{k_3}\rangle_{\rm div} \propto \frac{\lambda_3}{1+i\lambda_2 D_{k_1}(0,0)}\,.$$

This divergence will thus be absorbed if the coupling λ_3 is renormalized as

$$\lambda_3(\mu) = \frac{\lambda_3(\Lambda)}{1+\frac{\lambda_2(\Lambda)}{2\pi\alpha}\log\frac{\Lambda}{\mu}}\,, \tag{4.6}$$

and so

$$\beta_3(\Lambda) = \beta_3(\mu) + \frac{\lambda_3(\Lambda)\lambda(\Lambda)}{2\pi\alpha+\lambda_2(\Lambda)\log\frac{\Lambda}{\mu}}\log\frac{\Lambda}{\mu}\,, \tag{4.7}$$

Notice that this renormalization of λ_3 is precisely the one that ensures the renormalized coupling β_3 in eq. (4.7) to be independent of the four-momentum k. After renormalization, the quantity $\left(\beta_3 - i\lambda\lambda_3\tilde{D}_k(0,0)\right)$ is therefore finite

$$\mu\partial_\mu\left(\beta_3 - i\lambda\lambda_3\tilde{D}_k(0,0)\right) = 0 \tag{4.8}$$

which corresponds to the following flows for β_3 and λ_3

$$\mu\frac{{\rm d}\lambda_3(\mu)}{{\rm d}\mu} = \frac{\lambda_2(\mu)\lambda_3(\mu)}{2\pi\alpha} \quad\text{and}\quad \mu\frac{{\rm d}\beta_3(\mu)}{{\rm d}\mu} = \frac{\lambda(\mu)\lambda_3(\mu)}{2\pi\alpha}\,. \tag{4.9}$$

Once again, this tree-level renormalization leads to a perfectly well-defined notion of the three-point functions $G^{\chi\chi\chi}$ and $G^{\phi\chi\chi}(r)$, as long as r is not evaluated on the brane and we work outside the coincidence limit.

Furthermore, this renormalization of these coupling constants β_3 and λ_3 also ensures that the two additional three-point functions $\langle\phi(r_1)\phi(r_2)\chi\rangle$ and $\langle\phi(r_1)\phi(r_2)\phi(r_3)\rangle$ are finite

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(for $r_i > 0$). Indeed, we have, for $\sum_{i=1}^{3} \mathbf{k}_i = 0$,

$$
\begin{aligned}
G^{\phi\phi\chi}{}_{k_1,k_2,k_3}(r_1,r_2) &= \langle \phi_{k_1}(r_1)\phi_{k_2}(r_2)\chi_{k_3}\rangle \\
&= -i\Big(6\beta_3 G^{\phi\chi}_{k_1}(r_1)G^{\phi\chi}_{k_2}(r_2)G^{\chi\chi}_{k_3} + 2\lambda_3 G^{\phi\chi}_{k_1}(r_1)G^{\phi\chi}_{k_2}(r_2)G^{\phi\chi}_{k_3}(0) \\
&\qquad + 2\lambda_3\left(G^{\phi\chi}_{k_1}(r_1)G^{\phi\phi}_{k_2}(r_2,0) + (1\leftrightarrow 2)\right)G^{\chi\chi}_{k_3}\Big) \\
&= -2iG^{\chi\chi}_{k_3}\prod_{i=1}^{2}G^{\phi\chi}_{k_i}(r_i)\left(3\beta_3(\Lambda) - \lambda_3(\Lambda)\frac{i\lambda(\Lambda)D_{k_3}(0,0)}{1+i\lambda_2(\Lambda)D_{k_3}(0,0)} - \sum_{i=1}^{2}\frac{\lambda_3(\Lambda)}{i\lambda(\Lambda)H_{k_i}(\Lambda)}\right) \\
&= -2iG^{\chi\chi}_{k_3}\prod_{i=1}^{2}G^{\phi\chi}_{k_i}(r_i)\Bigg(3\beta_3(\mu) - \lambda_3(\mu)\sum_{i=1}^{3}\frac{\lambda(\mu)}{1+\frac{\lambda_2(\mu)}{2\pi\alpha}\log\frac{\mu}{k}}\frac{1}{2\pi\alpha}\log\frac{\mu}{k} \\
&\qquad + \sum_{i=1}^{2}\frac{\lambda_3(\mu)}{1+\frac{\lambda_2(\mu)}{2\pi\alpha}\log\frac{\mu}{k}}\frac{D_{k_i}(r_i,0)}{G^{\phi\chi}_{k_i}(r_i)}\Bigg),
\end{aligned}
$$

which, in terms of the renormalized coupling constants, is clearly finite. The last three-point function can be expressed in a similar way:

$$
\begin{aligned}
G^{\phi\phi\phi}_{k_1,k_2,k_3}(r_1,r_2,r_3) &= \langle \phi_{k_1}(r_1)\phi_{k_2}(r_2)\phi_{k_3}(r_3)\rangle \\
&= -i\left(\prod_{i=1}^{3}G^{\phi\chi}_{k_i}(r_i)\right)\left(6\beta_3 + 2\lambda_3\sum_{i=1}^{3}\frac{G^{\phi\phi}_{k_i}(r_i,0)}{G^{\phi\chi}_{k_i}(r_i)}\right) \\
&= -2i\left(\prod_{i=1}^{3}G^{\phi\chi}_{k_i}(r_i)\right)\sum_{i=1}^{3}\Bigg[\beta_3(\mu) - \lambda_3(\mu)\frac{\lambda(\mu)}{1+\frac{\lambda_2(\mu)}{2\pi\alpha}\log\frac{\mu}{k}_i}\frac{1}{2\pi\alpha}\log\frac{\mu}{k} \\
&\qquad + \frac{\lambda_3(\mu)}{1+\frac{\lambda_2(\mu)}{2\pi\alpha}\log\frac{\mu}{k}_i}\frac{D_{k_i}(r_i,0)}{G^{\phi\chi}_{k_i}(r_i)}\Bigg],
\end{aligned}
$$

and is therefore also completely finite. This result is already non-trivial as it stands, since all four three-point functions have been made finite by simple tree-level renormalization of the two coupling constants β_3 and λ_3. We can however push this analysis even a step further by studying the implications for the four-point functions as well as the loop corrections to the two-point functions.

4.2 Four-point functions

The classical contributions to the four-point function $\langle\chi^4\rangle$ are symbolically represented in figure 5.

Summing these diagrams, we get for $\sum_{i=1}^{4}\mathbf{k_i} = 0$,

$$
\begin{aligned}
G^{\chi\chi\chi\chi}_{k_1,k_2,k_3,k_3} &= \langle \chi_{k_1}\chi_{k_2}\chi_{k_3}\chi_{k_4}\rangle \\
&= \left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right)\Bigg[(-i)4!\beta_4 + \frac{(-i)^2}{2!}\sum_{\mathrm{perm_u}}2\Big\{6^2\beta_3^2G^{\chi\chi}_{k_u} + 2.12\beta_3\lambda_3G^{\phi\chi}_{k_u}(0) \qquad (4.10) \\
&\qquad + 12G^{\chi\chi}_{k_u}\sum_{i=1}^{4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}} + 4\lambda_3^2G^{\phi\phi}_{k_u}(0,0) + 4\lambda_3^2G^{\phi\chi}_{k_u}(0)\sum_{i=1}^{4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}} \\
&\qquad + 4\lambda_3^2G^{\chi\chi}_{k_u}\sum_{i=u_1,u_2}\sum_{j=u_3,u_4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}}\frac{G^{\phi\chi}_{k_j}(0)}{G^{\chi\chi}_{k_j}}\Big\}\Bigg],
\end{aligned}
$$

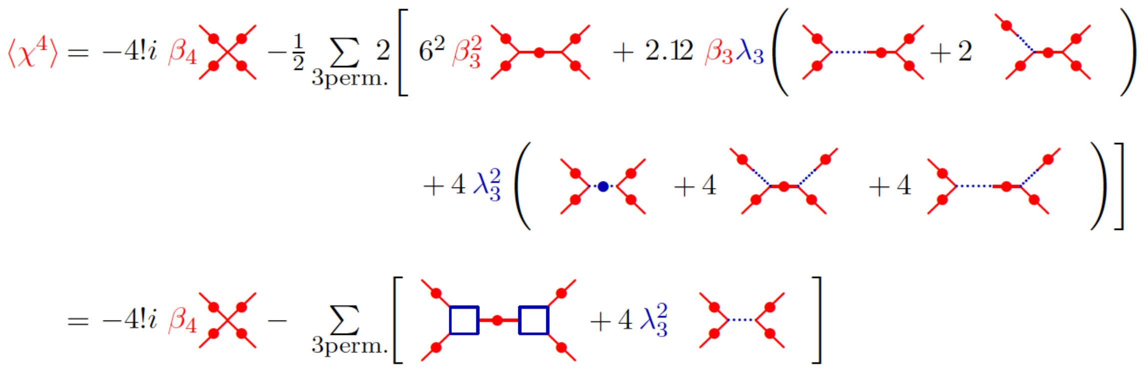

Figure 5: Classical contributions to the four-point functions. The second term on the bottom line is finite, but the third diagram involves a divergent piece proportional to $\lambda_3^2 \tilde{D}_k(0,0)$ which needs to be absorbed into the coupling β_4.

where we recall again that the factor $(-i)$ and $(-i)^2/2!$ arise from the expansion to first order (resp. second order) of $e^{-i\int d^4x\,\mathcal{H}^{\mathrm{int}}_{\chi\phi}}$. The sum over perm$_u$ is the one over the three permutations $u = \{(1234),\,(1324)\,,(1423)\}$, for which $k_u = k_{u_1} + k_{u_2} = \{k_1+k_2,\, k_1+k_3,\, k_1+k_4\}$.

We now recall that the product of the two three-point functions $\langle\chi_{k_1}\chi_{k_2}\chi_{k_u}\rangle\langle\chi_{k_3}\chi_{k_4}\chi_{k_u}\rangle$ is *finite* (after appropriate renormalization of the couplings β_3 and λ_3 in (4.6)) and is given by

$$
\begin{aligned}
\mathcal{A}_u &= \langle\chi_{k_{u_1}}\chi_{k_{u_2}}\chi_{k_u}\rangle\langle\chi_{k_{u_3}}\chi_{k_{u_4}}\chi_{k_u}\rangle \\
&= -\,(G^{\chi\chi}_{k_u})^2\left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right)\left(6\beta_3+2\lambda_3\sum_{i=u_1,u_2,u}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}}\right)\left(6\beta_3+2\lambda_3\sum_{j=u_3,u_4,u}\frac{G^{\phi\chi}_{k_j}(0)}{G^{\chi\chi}_{k_j}}\right) \\
&= -\Bigg(6^2\beta_3^2G^{\chi\chi}_{k_u}+12\beta_3\lambda_3G^{\chi\chi}_{k_u}\sum_{i=1}^{4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}}+24\beta_3\lambda_3G^{\phi\chi}_{k_u}(0)+4\lambda_3^2G^{\phi\chi}_{k_u}(0)\sum_{i=1}^{4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}} \\
&\qquad +4\lambda_3^2G^{\chi\chi}_{k_u}\sum_{i=u_1,u_2}\sum_{j=u_3,u_4}\frac{G^{\phi\chi}_{k_i}(0)}{G^{\chi\chi}_{k_i}}\frac{G^{\phi\chi}_{k_j}(0)}{G^{\chi\chi}_{k_j}}+4\lambda_3^2\frac{G^{\phi\chi}_{k_u}(0)G^{\phi\chi}_{k_u}(0)}{G^{\chi\chi}_{k_u}}\Bigg)G^{\chi\chi}_{k_u}\left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right).
\end{aligned}
$$

As shown symbolically in figure 5, after appropriate recombination of these different contributions, we can reexpress this four-point function as a finite product of these two renormalized three-point function plus a divergent piece which fixes the renormalization of the coupling β_4:

$$
\begin{aligned}
G^{\chi\chi\chi\chi}_{k_1,k_2,k_3,k_4} &= \sum_{\mathrm{perm_u}}\left\{\frac{1}{G^{\chi\chi}_{k_u}}\mathcal{A}_u+4(-i)^2\lambda_3^2\left[G^{\phi\phi}_{k_u}(0,0)-\frac{G^{\phi\chi}_{k_u}(0)G^{\phi\chi}_{k_u}(0)}{G^{\chi\chi}_{k_u}}\right]\left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right)\right\} \\
&\quad +4!(-i)\beta_4\left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right) \\
&= \sum_{\mathrm{perm_u}}\left\{\frac{1}{G^{\chi\chi}_{k_u}}\mathcal{A}_u+\left[4(-i)^2\lambda_3^2\tilde{D}_{k_u}(0,0)+(-i)\frac{4!}{3}\beta_4\right]\left(\prod_{i=1}^{4}G^{\chi\chi}_{k_i}\right)\right\}. \qquad (4.11)
\end{aligned}
$$

All terms in the previous expressions are finite apart from the ones in square brackets. The divergence of this term can once again be absorbed into β_4 using the following appropriate renormalization

$$\begin{aligned}\beta_4(\Lambda) - \frac{i}{2}\lambda_3^2(\Lambda)\tilde{D}_k(0,0) &= \beta_4(\Lambda) - \frac{\lambda_3^2(\Lambda)}{4\pi\alpha}\frac{\log\frac{\Lambda}{k}}{1+\frac{\lambda_2(\Lambda)}{2\pi\alpha}\log\frac{\Lambda}{k}} \\ &= \beta_4(\mu) - \frac{\lambda_3^2(\mu)}{4\pi\alpha}\frac{\log\frac{\mu}{k}}{1+\frac{\lambda_2(\mu)}{2\pi\alpha}\log\frac{\mu}{k}}\,, \end{aligned} \tag{4.12}$$

i.e. the renormalized coupling β_4 must flow as

$$\mu\partial_\mu\beta_4(\mu) = \frac{\lambda_3^2(\mu)}{4\pi\alpha}. \tag{4.13}$$

In other words, as soon as cubic interactions between the bulk and the brane field are introduced, a quartic interaction for the brane field is spontaneously generated classically. This is familiar for standard EFT.

We can also carefully check using exactly the same technique as previously that this renormalization of the coupling β_4 also ensures that all remaining four-point functions $\langle\phi\chi^3\rangle$, $\langle\phi^2\chi^2\rangle$, $\langle\phi^3\chi\rangle$ and $\langle\phi^4\rangle$ are completely finite classically (provided the bulk field is evaluated away from the brane),

$$\begin{aligned}
G^{\phi\chi\chi\chi}_{k_1,k_2,k_3,k_4}(r) &= \sum_{\mathrm{perm_u}}\Bigg\{\frac{1}{G^{\chi\chi}_{k_u}}G^{\phi\chi\chi}_{k_{u_1},k_{u_2},k_u}(r)G^{\chi\chi\chi}_{k_{u_3},k_{u_4},k_u} \\
&\qquad -4\left[\lambda_3^2\tilde{D}_{k_u}(0,0)+2i\beta_4\right]G^{\phi\chi}_{k_1}(r)\left(\prod_{i=2}^4 G^{\chi\chi}_{k_i}\right)\Bigg\} \\
G^{\phi\phi\chi\chi}_{k_1,k_2,k_3,k_4}(r_1,r_2) &= \frac{1}{G^{\chi\chi}_{k_1+k_2}}G^{\phi\phi\chi}_{k_1,k_2,k_1+k_2}(r_1,r_2)G^{\chi\chi\chi}_{k_3,k_4,k_1+k_2} \\
&\quad +\frac{1}{G^{\chi\chi}_{k_1+k_3}}G^{\phi\chi\chi}_{k_1,k_3,k_1+k_3}(r_1)G^{\phi\chi\chi}_{k_2,k_4,k_1+k_3}(r_2)+(3\leftrightarrow 4) \\
&\quad -\Bigg(4\left[\lambda_3^2\tilde{D}_{k_1+k_3}(0,0)+2i\beta_4\right]+(3\leftrightarrow 4) \\
&\qquad +4\left[\lambda_3^2\tilde{D}_{k_1+k_2}(0,0)+2i\beta_4\right]\Bigg)\left(\prod_{i=1}^2 G^{\phi\chi}_{k_i}(r_i)\right)\left(\prod_{i=3}^4 G^{\chi\chi}_{k_i}\right) \\
G^{\phi\phi\phi\chi}_{k_1,k_2,k_3,k_4}(r_1,r_2,r_3) &= \sum_{\mathrm{perm_u}}\Bigg\{\frac{1}{G^{\chi\chi}_{k_u}}G^{\phi\phi\chi}_{k_{u_1},k_{u_2},k_u}(r_{u_1},r_{u_2})G^{\phi\chi\chi}_{k_{u_3},k_4,k_u}(r_{u_3}) \\
&\qquad -4\left[\lambda_3^2\tilde{D}_{k_u}(0,0)+2i\beta_4\right]\left(\prod_{i=1}^3 G^{\phi\chi}_{k_i}(r_i)\right)G^{\chi\chi}_{k_4}\Bigg\} \\
G^{\phi\phi\phi\phi}_{k_1,k_2,k_3,k_4}(r_1,r_2,r_3,r_4) &= \sum_{\mathrm{perm_u}}\Bigg\{\frac{1}{G^{\chi\chi}_{k_u}}G^{\phi\phi\chi}_{k_{u_1},k_{u_2},k_u}(r_{u_1},r_{u_2})G^{\phi\phi\chi}_{k_{u_3},k_{u_4},k_u}(r_{u_3},r_{u_4}) \\
&\qquad -4\left[\lambda_3^2\tilde{D}_{k_u}(0,0)+2i\beta_4\right]\left(\prod_{i=1}^4 G^{\phi\chi}_{k_i}(r_i)\right)\Bigg\}.
\end{aligned}$$

Each of these four-point functions introduces a combination of the couplings which is completely finite once they have been renormalized as specified previously (*i.e.* $\lambda_3^2 + \tilde{D}_k(0,0) + 2i\beta_4$ is finite). This non-trivial check ensures that our proposal makes sense at least at the classical level up to the four-point function. Before discussion the general renormalizability of this theory for higher point functions, we present in what follows an insight into the situation at the quantum level, *i.e.* when loops are taken into account.

4.3 Loops

At the loop level, we expect from standard field theory in four dimensions, that UV divergences will arise from the momentum integral over the loop. However no further divergences arise from the codimension-two nature of the theory, and the counterterms required to absorb the divergences are thus the usual one of four-dimensional field theory.

To start with, we concentrate on the two-point function of the brane field χ. At first order in loops, (second order for β_3 and λ_3) one has

$$\langle\chi\chi\rangle_{1\_\text{loop}} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\, \mathcal{I}_{\text{loop}}(p,k)\; G_k^{\chi\chi} G_k^{\chi\chi}\,, \tag{4.14}$$

with the integrand $\mathcal{I}_{\text{loop}}(p,k)$ being the sum over the different loop configurations:

$$\begin{aligned}\mathcal{I}_{\text{loop}}(p,k) =&\; \frac{(-i)^2}{2!} G_p^{\chi\chi} G_{k-p}^{\chi\chi} \left(6\beta_3 - 2i\lambda_3\lambda\left(\tilde{D}_k + \tilde{D}_p + \tilde{D}_{k-p}\right)\right)^2 \\ &\frac{(-i)^2}{2!} G_k^{\chi\chi} G_0^{\chi\chi} \left(6\beta_3 - 2i\lambda_3\lambda\left(\tilde{D}_k + \tilde{D}_p + \tilde{D}_0\right)\right)^2 \\ &+4\lambda_3^2\left(G_p^{\chi\chi}\tilde{D}_{k-p} + G_{k-p}^{\chi\chi}\tilde{D}_p + G_p^{\chi\chi}\tilde{D}_0\right) + (-i)\frac{4!}{2}\beta_4 G_p^{\chi\chi}\,,\end{aligned} \tag{4.15}$$

where for simplicity we have used the notation $\tilde{D}_k \equiv \tilde{D}_k(0,0)$. This expression can be most easily interpreted as finite products of three-point functions and extra terms as symbolized in figure 6

$$\begin{aligned}\mathcal{I}_{\text{loop}}(p,k) =&\; -\frac{1}{2}\frac{(G_{k,p,k-p}^{\chi\chi\chi})^2}{G_p^{\chi\chi}G_{k-p}^{\chi\chi}(G_k^{\chi\chi})^2} - \frac{1}{2}\frac{G_{k,k,0}^{\chi\chi\chi}G_{p,p,0}^{\chi\chi\chi}}{G_p^{\chi\chi}G_0^{\chi\chi}(G_k^{\chi\chi})^2} \\ &-2\left[\lambda_3^2\tilde{D}_p + 2i\beta_4\right]G_{k-p}^{\chi\chi} - 2\left[\lambda_3^2\tilde{D}_{k-p} + 2i\beta_4\right]G_p^{\chi\chi} \\ &-2\left[\lambda_3^2\tilde{D}_0 + 2i\beta_4\right]G_p^{\chi\chi}\,.\end{aligned} \tag{4.16}$$

To clarify the discussion, we denote by Λ, the cutoff scale associated with the codimension-two brane thickness, or equivalently with the integration over the momentum along the extra dimensions, while Δ designates the standard four-dimensional momentum cut-off scale, *i.e.* in (4.16) the loop integration is cutoff at the scale Δ: $\int \mathrm{d}^4 p \sim \int_0^\Delta \mathrm{d}p p^3$. Clearly these two scales could be associated with one another, simply representing the scale at which UV physics becomes important. However for sake of simplicity, we distinguish for now between these two quantities and assume that in general they could be different. In this scenario, the couplings are then flowing along two distinct directions Λ and Δ, and we focus our attention on the flow along the Λ direction.

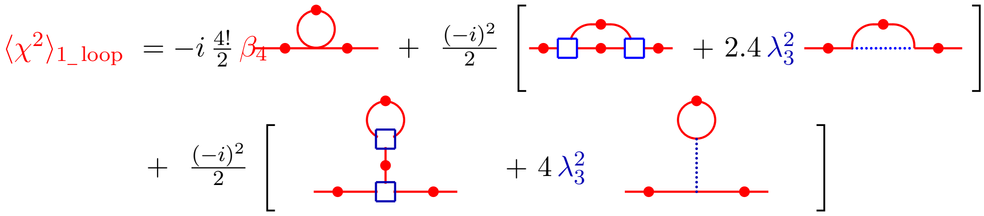

Figure 6: One-loop corrections to the two-point function. On both lines, the second diagram is finite, while the first and third diagrams contain logarithmic divergences that will not cancel each other.

On simple dimension grounds, we expect that the loop integral over p will diverge logarithmically, and the one-loop contribution to the two-point function has thus a cutoff dependence of the form $\log\Delta$ which should be absorbed by introduction of a mass counterterm of the form $\delta m^2 \sim \log\Delta$. This is standard procedure in four-dimensional field theory.

We are however more concern here on the dependence of the other cutoff Λ. λ_3 and β_3 have been renormalized in (4.7), such that the three-point function $G^{\chi\chi\chi}$ is finite so the first line of (4.16) is clearly finite. Although the rest of expression (4.16) includes divergent terms of the form $\tilde{D}_k(0,0)$, the combination involved $\left[\lambda_3^2\tilde{D}_k + 2i\beta_4\right]$ is precisely the combination that appeared in the expression of four-point function (4.11), (4.12), and is thus also finite.

Notice furthermore that the one-loop correction to the two remaining two-point functions $\langle\phi(r)\chi\rangle_{1\_\mathrm{loop}}$ and $\langle\phi(r)\phi(r')\rangle_{1\_\mathrm{loop}}$ will be also be finite in the thin-brane limit (once the loop divergences associated with Δ have been taken care of), as shown explicitly for the four-point function.

However a non-trivial feature emerges from the computation of the one-loop corrections. The expression (4.16) involves terms of the form $\tilde{D}_0(0,0)$ which also diverges logarithmically, but this time this divergence is instead associated with a IR behaviour. In this toy-model, the origin of this IR divergence is related to the fact that the bulk field ϕ is massless, but would disappear as soon as a small mass m_ϕ^2 was introduced. However, if the bulk field is to mimic the graviton, this field should remain massless. Physically, such IR divergences can be removed in the same way as in quantum electrodynamics, see ref. [27].

4.4 Is the theory renormalizable?

To complete this section, we argue that this scalar field toy-model is renormalizable, provided that only relevant and marginal operators are considered. Since no coupling of the form $\beta_N\chi^N$, $(N \geq 5)$ is introduced, any further N-point function will necessarily be composed of only reducible diagrams and will thus be expressible in terms of lower-dimensional n-point functions ($n \leq 4$.) Since we have shown that all of these n-point functions are finite at the classical level, any further N-point function will thus automatically be finite, without any further counterterms.

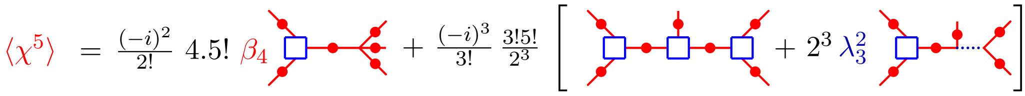

Figure 7: Classical contributions to the five-point function. The second diagram is finite, while the first and third diagrams contains logarithmic divergences that compensate each other. This five-point function is therefore finite.

We make this argument more concrete by exploring the five-point function, and showing explicitly that it remains finite in the thin-brane limit if β_4 is renormalized as in (4.12). A completely general argument for an arbitrary N-point function can be found in appendix B. In particular we show that the same will remains true at the level of any N-point function. The theory will thus be renormalizable, as one can expect from standard four-dimensional field theory intuition.

The contributions to the five-point function are symbolized in figure 7.

Summing these diagrams, we obtain

$$
\begin{aligned}
G^{\chi^5}_{k_1,\dots,k_5} &= \frac{(-i)^3}{3!}\,\frac{3!5!}{2^3}\,\frac{G^{\chi\chi\chi}_{k_1,k_2,(k_1+k_2)}G^{\chi\chi\chi}_{k_3,(k_1+k_2),(k_4+k_5)}G^{\chi\chi\chi}_{(k_4+k_5),k_4,k_5}}{G^{\chi\chi}_{k_1+k_2}G^{\chi\chi}_{k_4+k_5}} \\
&\quad + \left(\frac{(-i)^3}{3!}\,\frac{3!5!}{2^3}\,2^3\lambda_3^2\tilde{D}_{k_1+k_2} + \frac{(-i)^2}{2!}\,4.5!\,\beta_4\right) G^{\chi\chi\chi}_{(k_4+k_5),k_4,k_5}\prod_{i=1}^{3}G^{\chi\chi}_{k_i} \\
&= 5!i\left[\frac{G^{\chi\chi\chi}_{k_1,k_2,(k_1+k_2)}G^{\chi\chi\chi}_{k_3,(k_1+k_2),(k_4+k_5)}}{G^{\chi\chi}_{k_1+k_2}G^{\chi\chi}_{k_4+k_5}}\right. \\
&\qquad \left. + \left(\lambda_3^2\tilde{D}_{k_1+k_2} + 2i\beta_4\right)\prod_{i=1}^{3}G^{\chi\chi}_{k_i}\right] G^{\chi\chi\chi}_{(k_4+k_5),k_4,k_5}\,,
\end{aligned}
\tag{4.17}
$$

where for simplicity we have used a specific momentum configuration, but the counting takes in account all possible permutations.

The first line of the pervious expression is trivially finite, while the second line is finite only if the terms proportional to β_4 and $\lambda_3^2\tilde{D}_k(0,0)$ contribute with appropriate coefficients. As can be seen in the third line of this expression, the contribution from these terms is also finite as β_4 is renormalized precisely so as to have $\lambda_3^2\tilde{D}_k + 2i\beta_4$ finite. The five-point function is therefore finite at the classical level and no counterterms ought to be added. This result will remain valid for any other five-point functions (*i.e.* including the ones with bulk external fields $\langle\phi(r)\chi\chi\chi\chi\rangle$, etc.), as well as for any higher N-point function and their loop corrections, (see appendix B). This represents a highly non-trivial check and leads to the conclusion that the theory is completely renormalizable against divergences associated with the codimension-two source.

5. Electromagnetism on a codimension-two brane

In this section, we consider the more physical scenario of a massless gauge field A_μ confined to the brane and coupled to gravity. This represents a more realistic framework to study the coupling between gravity and electromagnetism. We start with the following six-dimensional action

$$S^{(\mathrm{em})} = \int \mathrm{d}^6x\sqrt{-g}\left[\frac{1}{2\kappa^2}R^{(6)} - \delta^2(y)\,\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right], \tag{5.1}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In what follows we work at linear order in perturbations around a flat conical background:

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}x^a\mathrm{d}x^b = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + (\eta_{\mu\nu} + h_{\mu\nu})\,\mathrm{d}x^\mu\mathrm{d}x^\nu\,, \tag{5.2}$$

where we work in de Donder gauge, $h^\mu_{\nu,\mu} = \frac{1}{2}h^\mu_{\mu,\nu}$. Since the stress-energy for radiation is transverse, we will have $h^\mu{}_\mu = 0$. The Einstein's equations impose

$$\begin{aligned} G_{\mu\nu} &= \kappa^2 T^{\mathrm{em}}_{\mu\nu} \\ -\frac{1}{2}\Box^{(6)}h_{\mu\nu} &= -\kappa^2\frac{\delta(r)}{2r\pi\alpha}\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}F^2\eta_{\mu\nu}\right). \end{aligned} \tag{5.3}$$

Using results from the previous sections, we know that $\tilde{h}_{\mu\nu}$ will diverge logarithmically when evaluated at $r = 0$

$$h_{\mu\nu}(0) = \frac{\kappa^2}{4\pi\alpha}\left(\Gamma + \log\frac{k\epsilon}{2}\right)\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}F^2\eta_{\mu\nu}\right), \tag{5.4}$$

where Γ is the Euler number and $\epsilon \to 0$ represents the thin-brane limit. This will affect the equation of motion for the photon:

$$\begin{aligned} \nabla_\mu F^{\mu\nu} &= \left(\eta^{\mu\tilde{\mu}} - h^{\mu\tilde{\mu}}\right)\left(\eta^{\nu\tilde{\nu}} - h^{\nu\tilde{\nu}}\right)\left(\partial_\mu F_{\tilde{\mu}\tilde{\nu}} - \Gamma^\alpha_{\mu\tilde{\mu}}F_{\alpha\tilde{\nu}} - \Gamma^\alpha_{\mu\tilde{\nu}}F_{\tilde{\mu}\alpha}\right) \\ &= \partial_\mu F^{\mu\nu} - \partial_\mu\left(h^{\alpha\nu}F^\mu{}_\alpha - h^{\alpha\mu}F^\nu{}_\alpha\right) = 0\,, \end{aligned} \tag{5.5}$$

where in the second line, all index raising is performed with respect to the background flat metric $\eta^{\alpha\beta}$. We remember that in the previous expression, $h_{\alpha\beta}$ represents the induced value of the metric perturbation evaluated on the brane and thus diverges logarithmically in the thin-brane limit. If this was the end of the story, then photons would be very sensitive to the brane thickness ϵ even at low-energy. However, we have learned from section 3, that as soon as a coupling λ is introduced between brane and bulk fields, this spontaneously generates a mass term m^2 for the brane field at the classical level. The situation is no different here, and the logarithmic divergence of h on the brane will spontaneously generate F^4 terms on the brane. More precisely, let us consider the Euler-Heisenberg brane action

$$S_{(\mathrm{brane})} = -\int \mathrm{d}x^4\sqrt{-q}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\gamma_1}{8}\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \frac{\gamma_2}{8}F_{\mu\nu}F^\mu{}_\alpha F_\beta{}^\alpha F^{\beta\nu}\right], \tag{5.6}$$

so that the Maxwell's equations (5.5) are modified to

$$\partial_\mu F^{\mu\nu} - \partial_\mu\left(h^{\alpha\nu}F^\mu{}_\alpha - h^{\alpha\mu}F^\nu{}_\alpha\right) - \gamma_1\partial_\mu\left(F^2F^{\mu\nu}\right) - \gamma_2\partial_\mu\left(F^\mu{}_\alpha F^\alpha{}_\beta F^{\nu\beta}\right) = 0. \tag{5.7}$$

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Substituting the expression (5.4) for the perturbed metric on the brane, we obtain

$$\partial_\mu F^{\mu\nu} + \left(\frac{\kappa^2}{8\pi\alpha}\left(\Gamma + \log\frac{k\epsilon}{2}\right) - \gamma_1\right)\partial_\mu\left(F^2 F^{\mu\nu}\right) - \left(\frac{\kappa^2}{2\pi\alpha}\left(\Gamma + \log\frac{k\epsilon}{2}\right) + \gamma_2\right)\partial_\mu\left(F^\mu{}_\alpha F^\alpha{}_\beta F^{\nu\beta}\right) = 0\,. \tag{5.8}$$

The divergence of the graviton can thus be absorbed in the two couplings γ_1 and γ_2:

$$\gamma_1(\mu) = \gamma_1(\epsilon) + \frac{\kappa^2}{8\pi\alpha}\log\frac{\mu}{\epsilon} \qquad \text{and} \qquad \gamma_2(\mu) = \gamma_2(\epsilon) - \frac{\kappa^2}{2\pi\alpha}\log\frac{\mu}{\epsilon}\,, \tag{5.9}$$

leading to the following RG flows

$$\mu\partial_\mu\gamma_1(\mu) = -\frac{1}{4}\,\mu\partial_\mu\gamma_2(\mu) = \frac{\kappa^2}{8\pi\alpha}\,. \tag{5.10}$$

The generation of F^4 terms on the brane at the classical level ensures that a photon confined to a codimension-two brane and interacting with a gravitational wave will not evolve in a regularization-dependent way. In particular, this mechanisms ensures that at low-energy, there is a well defined thin-brane limit description of the codimension-two brane. Of course once, F^4 terms are introduced, they will in turn introduce divergences on the brane, which should be absorbed with higher order terms. The proper finite theory will hence include a infinite series.

6. Localized kinetic terms

As a last intriguing physical implication, we consider in this section the consequences for localized kinetic terms on the brane. Localized kinetic terms are of importance when considering brane-induced Einstein-Hilbert terms, where the action is typically of the form

$$S = \frac{M_{(d)}^{d-2}}{2}\int \mathrm{d}^d x\sqrt{-g_d}\,R_{(d)} + \int \mathrm{d}^4 x\sqrt{-g_4}\left(\frac{M_{(4)}^2}{2}R_{(4)} + \mathcal{L}_{\mathrm{matter}}\right). \tag{6.1}$$

The induced Einstein-Hilbert term $R_{(4)}$ term is expected to be spontaneously generated at the quantum level, and represents a natural mechanism to localize gravity on a four-dimensional surface when the extra dimensions are flat and infinite [14, 28]. Such models also represent a physical realization of the degravitation process (see ref. [13]), since in such scenarios gravity becomes fully higher-dimensional at long wavelengths, hence providing a potential explanation for the observed value of the cosmological constant. Such models are also enriched with an additional interesting feature namely the possibility of having self-accelerating branches [29] (see refs. [30] for ghost-free realizations).

Although such models are usually considered in the context of one large extra dimension, a simultaneous resolution of the Hierarchy problem usually requires at least two extra dimensions [7], and the degravitation observed in the presence of only one extra dimension is only marginal. Understanding this scenario in the presence of two

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extra dimensions is therefore an important next step. In what follows, we examine the consequences of such kinetic terms in a scalar field toy-model.

We consider a massless scalar field ϕ living in a six-dimensional flat space-time with induced kinetic terms on a codimension-two brane

$$S = -\int \mathrm{d}^6 x \left[\frac{1}{2}(\partial_a \phi)^2 + \delta^2(y)\,\phi\, f(\Box)\,\phi\right], \tag{6.2}$$

where $\Box$ represents the four-dimensional d'Alembertian $\Box = \partial^\mu \partial_\mu$. In order to recover on the brane the standard Klein-Gordon equation for the scalar field in the infrared, $\left(\Box + m^2\right)\phi = 0$, we require that only positive powers of $\Box$ be present in f. In particular we write

$$f(\Box) = \sum_{n\geq 0} c_n \left(\ell^2 \Box\right)^n, \tag{6.3}$$

where ℓ is an arbitrary length scale (we recall that in this six-dimensional formalism, ϕ has dimension mass squared and thus f ought to be dimensionless). In particular, c_0 represents the dimensionless coupling λ_2 that was considered in section 3. From that section, we know that the scalar field will be well-defined away from the brane only if the induced couplings on the brane (in this case the function f) are renormalized and flow as

$$\mu\partial_\mu f(\Box) = \frac{1}{2\pi\alpha} f^2(\Box)\,. \tag{6.4}$$

In terms of the coefficients c_n, this implies

$$\mu\partial_\mu c_n(\mu) = \frac{1}{2\pi\alpha}\sum_{u=0}^{n} c_{n-u}(\mu) c_u(\mu)\,. \tag{6.5}$$

This has important consequences for these kind of theories, in particular, we are not free to choose a brane induced function of the form $f(\Box) = (m^2 + \Box)$, as higher curvature terms will spontaneously be generated at the tree-level. As soon as a mass term c_0 and kinetic term c_1 are present, all the other couplings c_n will flow in a non-trivial way. The solution for the two first terms is of the form

$$c_0(\mu) = \frac{\bar{c}_0}{1 - \frac{1}{2\pi\alpha}\bar{c}_0 \log\mu} \tag{6.6}$$

$$c_1(\mu) = \frac{\bar{c}_1}{\left(2\pi\alpha - \bar{c}_0\log\mu\right)^2} = \beta c_0(\mu)^2\,, \tag{6.7}$$

where $\beta = \bar{c}_1/2\pi\alpha\bar{c}_0^2$ is a dimensionless parameter.

As an example, one can choose a particular solution of (6.5), for which

$$c_n(\mu) = \beta^n c_0(\mu)^{n+1}\,, \tag{6.8}$$

and so

$$f(\Box) = \frac{c_0(\mu)}{1 - \beta\, c_0(\mu)\Box}\,. \tag{6.9}$$

Notice that in this formalism, the function of the kinetic term is fixed by the renormalization conditions, and very few parameters can actually be tuned arbitrarily. In this sense this represents a much more satisfying candidate for theories of modified gravity than for instance $f(R)$ gravities, (see ref. [31] for a review on such theories). As a natural extension, one should understand the cosmology for such a scenario and possibly the different signatures which could allow for the discrimination of codimension-two models.

7. Conclusions

We have analyzed the coupling between bulk fields living in a six-dimensional flat space-time and brane fields confined onto a four-dimensional surface. Due to these couplings, logarithmic divergences that arise when evaluating the bulk field on the brane generically propagate into the brane field. In this paper, we have presented a consistent renormalization mechanism at tree and one-loop level that removes any divergences simultaneously in the brane field and bulk field when the latter is evaluated away from the brane. We have also shown that any five-point function is finite at the classical level without the addition of any further counterterm, and demonstrated that this remains true for any N-point function at any order in the loop expansion, thus proving the renormalizability of the theory. We also point out the presence of IR divergences in the loop diagrams which can be dealt with the same way as for quantum electrodynamics.

The same principle can be applied to more complex theories such as electromagnetism in curved space-time, for which the same prescription remains completely valid. In particular we show that at tree-level, the coupling of electromagnetism to gravity generates a quartic term for the form field in the action, giving rise to a Euler-Heisenberg Lagrangian which is usually only generated via quantum corrections.

As another physical application, we have also explored the consequences for localized kinetic terms which are relevant in scenarios such as the DGP model. In particular we show that as soon as a kinetic term are induced on the brane, one cannot prevent for the generation of an infinite series of higher order terms. This provides a natural modification of gravity on the brane which might have potential interesting signatures.

To our knowledge, this prescription is the only one to date capable of making sense of sources on codimension-two branes in a regularization-independent way and providing a way to derive the low-energy effective theory on such objects in a regime where interactions with the bulk cannot be ignored.

Since the main objective of this paper was the establishment of a consistent framework to study sources on codimension-two branes, implications for braneworld physics have only been superficially addressed. Armed with these new tools, extensions to more physical scenarios will however be of great importance. Understanding the relevance of our results for electromagnetism and theories with induced gravity terms were beyond the scope of this paper but will present interesting extensions. More realistic interactions from the standard model would also be intriguing, and in particular consequences to the Higgs physics in six-dimensional scenarios, such as the SLED, [32] should be understood in more detail.

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A. Most general second order counterterms

In what follows, we consider a free scalar field living on a flat six-dimensional spacetime with a conical singularity at $r = 0$. As shown in (2.1) and (3.20), the brane-brane propagator of this field diverges logarithmically in the thin-brane limit: $D_k(0,0) \sim \log \Lambda/k$ as $\Lambda \to \infty$. In order to make this quantity finite, one can try to include counterterms both in the bulk and the brane. Unlike in usual EFT, these counterterms are not added to make the interacting theory finite, but the classical free theory itself. We consider the following most general set of counterterms to renormalize the free theory (renormalization of the wave function and mass both in the bulk and brane)

$$S = -\int \mathrm{d}^6x \left[(1+Z_1)(\partial_a\phi)^2 + \frac{1}{2}M^2\phi^2 + \delta^{(2)}(y)\left(Z_2(\partial\phi)^2 + \frac{1}{2}\lambda_2\phi^2\right)\right]. \tag{A.1}$$

The propagator will now be instead

$$\begin{aligned} D_k^\Lambda(r,r') &= \sum_{n=-\infty}^{+\infty}\int_0 \frac{q\mathrm{d}q}{2\pi\alpha}\frac{e^{i\tilde{n}(\theta-\theta')}}{(1+Z_1(\Lambda))(q^2+k^2)+M^2(\Lambda)}J_{|\tilde{n}|}(qr)J_{|\tilde{n}|}(qr') \\ &= \sum_{n=-\infty}^{+\infty}\left[K_{|\tilde{n}|}\left(\sqrt{k^2+\mu^2}\,r\right)I_{|\tilde{n}|}\left(\sqrt{k^2+\mu^2}\,r'\right)\Theta(r-r')\right. \\ &\qquad \left. + (r\leftrightarrow r')\right]\frac{e^{i\tilde{n}(\theta-\theta')}}{2\pi\alpha(1+Z_1)}, \end{aligned} \tag{A.2}$$

where $\tilde{n} = n/\alpha$ and $\mu^2(\Lambda) = \frac{M^2(\Lambda)}{1+Z_1(\Lambda)}$. Because of the brane counterterms Z_2 and λ_2, this is however not the complete two-point function. The two-point function is obtained by summing over all the interactions with the bulk coupling. This gives rise to following "dressed" propagators

$$G_k(r,r') = D_k^\Lambda(r,r') - \frac{Z_2(\Lambda)k^2+\lambda_2(\Lambda)}{1+(Z_2(\Lambda)k^2+\lambda_2(\Lambda))D_k^\Lambda(0,0)}D_k^\Lambda(r,0)D_k^\Lambda(0,r') \tag{A.3}$$

$$G_k(r,0) = D_k^\Lambda(r,0)\left[1 - \frac{Z_2(\Lambda)k^2+\lambda_2(\Lambda)}{1+(Z_2(\Lambda)k^2+\lambda_2(\Lambda))D_k^\Lambda(0,0)}D_k^\Lambda(0,0)\right] \tag{A.4}$$

$$G_k(0,0) = D_k^\Lambda(0,0)\left[1 - \frac{Z_2(\Lambda)k^2+\lambda_2(\Lambda)}{1+(Z_2(\Lambda)k^2+\lambda_2(\Lambda))D_k^\Lambda(0,0)}D_k^\Lambda(0,0)\right]. \tag{A.5}$$

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Notice that in this approach, we no longer require the propagator $D_k^\Lambda(r,r')$ to be finite in the thin brane limit but require instead that the two-point function G_k between any two points (taken in the bulk or the conical tip) is finite *i.e.* Z_1, Z_2, M^2 and λ_2 should flow in such a way that $G_k(r,r'), G_k(r,0)$ and $G_k(0,0)$ are all finite. This implies that:

- $D_k^\Lambda(0,0)/D_k^\Lambda(r,0)$ should be finite in the limit $\Lambda \to \infty$ for any value of r,
- and similarly the quantity $\left[D_k^\Lambda(r,r') - D_k^\Lambda(r,0)D_k^\Lambda(0,r')/D_k^\Lambda(0,0)\right]$ should be finite for any r and r'.

It will therefore only be possible to make sense of the two-point function on the brane, if one can renormalize the wave function and the mass in such a way that the ratio $D_k^\Lambda(0,0)/D_k^\Lambda(r,0)$ is finite. From (A.2), we get

$$D_k^\Lambda(r,0) = \frac{1}{2\pi\alpha(1+Z_1(\alpha))} K_0\left(\sqrt{k^2+\mu^2(\Lambda)}\, r\right) , \tag{A.6}$$

and

$$D_k^\Lambda(0,0) = \lim_{\Lambda\to\infty} D_k^\Lambda(\Lambda^{-1},0) = \frac{1}{2\pi\alpha(1+Z_1(\alpha))} \log\frac{\Lambda}{\sqrt{k^2+\mu^2(\Lambda)}} . \tag{A.7}$$

It is therefore clear from these expressions that no matter how the wave function and the mass renormalization flow, the quantity $(D_k^\Lambda(r,0)/D_k^\Lambda(0,0))$ will never be finite in the thin brane limit:

$$\frac{D_k^\Lambda(0,0)}{D_k^\Lambda(r,0)} = \frac{\log\Lambda - \frac{1}{2}\log(k^2+\mu^2(\Lambda))}{K_0(\sqrt{k^2+\mu^2(\Lambda)}\, r)} \to \infty \quad \text{as} \quad \Lambda\to\infty, \quad \forall \;\; \mu(\Lambda) . \tag{A.8}$$

No local counterterm (quadratic in the field) will thus ever make the two-point function finite everywhere both in the bulk and the brane.

B. General N-point function

In this appendix, we demonstrate that the RG flows of the brane couplings m, β_3, β_4, λ, λ_2, λ_3, (3.29), (4.6), (4.7) and (4.12) is sufficient to make all N-point Green's functions finite in the thin-brane limit at any order in the loop expansion, hence justifying the renormalizability of the theory. We focus, in what follows, on divergences associated with the codimension-two nature of the theory and do not discuss loop divergences which can be renormalized the standard way.

To simplify, we present the argument for the N-point Green's functions having only brane field external legs χ. As seen in section 4, the generalization to an arbitrary number of bulk field legs ϕ is straight forward.

Defining the generating functional $G[J]$

$$G[J] = \int \mathcal{D}\left[\chi,\phi\right] \langle 0| e^{-i(\mathcal{H}^{\rm int}_{\chi\phi}+J\chi)}|0\rangle , \tag{B.1}$$

the N-point Green's functions are expressed by

$$G^{(N)}(x_1, \ldots, x_N) = \left. \frac{\delta^n G[J]}{\delta J(x_1) \cdots \delta J(x_N)} \right|_{J=0} . \tag{B.2}$$

Technically, we are only interested in the connected Green's functions which are generated by $G_c[J] = -i \log G[J]$. The connected N-point Green's function can thus be expressed in terms of the lower ones as

$$G_c^{(N)} = -i \left(\frac{G^{(N)}}{G^{(0)}} + \sum_{n=1}^{N-1} C_{N-1}^n \frac{G^{(n)}}{G^{(0)}} G_c^{(N-n)} \right) , \tag{B.3}$$

where $C_m^n = \begin{pmatrix} n \\ m \end{pmatrix}$ is the binomial coefficient.

We have previously demonstrated that all the connected tree-level Green's functions $G_c^{(n)}$ were finite for $n < 5$. In what follows, we show that this results remains true for any Green's function $G^{(N)}$, $N \geq 0$ at any order in the loop expansion, thus ensuring that all the connected Green's is finite for any arbitrary number of external legs.

The expression for the N-point function is

$$\begin{aligned} G^{(N)}(x_1, \ldots, x_N) = \sum_{n \geq 0} \frac{(-i)^n}{n!} \Big\langle \chi(x_1) \cdots \chi(x_N) \\ \times \prod_{i=1}^{n} \int \mathrm{d}y_i \big(\beta_3 \chi^3(y_i) + \beta_4 \chi^4(y_i) + \lambda_3 \phi(0, y_i) \chi^2(y_i)\big) \Big\rangle . \end{aligned}$$

Omitting the evaluation points x_i and y_i and remembering that every field is evaluated on the brane, we have

$$\begin{aligned} G^{(N)} &= \sum_{n \geq 0} \frac{(-i)^n}{n!} \left\langle \chi^N \left(\beta_3 \chi^3 + \beta_4 \chi^4 + \lambda_3 \phi \chi^2 \right)^n \right\rangle \\ &= \sum_{n \geq 0} \sum_{m=0}^{n} \sum_{k=0}^{m} \frac{(-i)^n}{n!} C_n^m C_m^k \beta_3^{m-k} \beta_4^{n-m} \lambda_3^k \left\langle \chi^{N-k+4n-m} \phi^k \right\rangle \end{aligned}$$

among these diagrams, some of them can connect two bulk fields together, generating a singular two-point function $G^{\phi\phi}(0,0)$. There can be α such connections, (with $0 \leq \alpha \leq k/2$), so that

$$G^{(N)} = \sum_{n \geq 0} \sum_{m=0}^{n} \sum_{k=0}^{m} \sum_{\alpha=0}^{k/2} \frac{(-i)^n}{n!} C_n^m C_m^k C_k^{2\alpha} \beta_3^{m-k} \beta_4^{n-m} \lambda_3^k \langle \chi^{N-k+4n-m} \phi^{k-2\alpha} \rangle \langle \phi^{2\alpha} \rangle .$$

We now consider the number of ways there is to connect the different fields together (we recall that for now we are interested in all the possible configurations, and do not restrict ourselves to the connected ones). For x fields $\langle \chi^x \rangle$, there is
$P_x = (x-1)(x-3) \cdots 3$ possible configurations if x is even, and no possible configurations if x is odd ($\langle \chi \rangle = 0$). There is therefore $P_{2\alpha}$ ways to connect the 2α bulk fields in $\langle \phi^{2\alpha} \rangle$.

To count the number of configurations in $\langle \chi^{N-k+4n-m}\phi^{k-2\alpha}\rangle$, we need to pick first of all the fields χ which will connect with the remaining $k-2\alpha$ fields ϕ. There is $(N-k+4n-m)!/(N-k+4n-m-(k-2\alpha))!$ such configurations and then $P_{N-k+4n-m-(k-2\alpha)}$ ways to connect the remaining χ together. Putting all this together, we therefore get

$$
\begin{aligned}
G^{(N)} = \sum_{n\geq 0}\sum_{m=0}^{n}\sum_{k=0}^{m}\sum_{\alpha=0}^{k/2}\Bigg(&\frac{(-i)^n}{n!}C_n^m C_m^k\ \beta_3^{m-k}\beta_4^{n-m}\lambda_3^k C_k^{2\alpha} \\
&\times \frac{(N-k+4n-m)!}{(N-2k+4n-m+2\alpha)!}P_{N-2k+4n-m+2\alpha}P_{2\alpha} \\
&\times\ (G^{\chi\chi})^{\frac{N-2k+4n-m+2\alpha}{2}}\left(G^{\chi\phi}(0)\right)^{k-2\alpha}\left(G^{\phi\phi}(0,0)\right)^{\alpha}\Bigg)
\end{aligned}
\tag{B.4}
$$

with

$$
P_x = \begin{cases} (x-1)(x-3)\cdots 3 = \frac{2n!}{2^n n!} & \text{if } x \text{ is even},\ x=2n>0, \\ 1 & \text{if } x=0, \\ 0 & \text{otherwise.}\end{cases}
\tag{B.5}
$$

We therefore notice that in the previous expression (B.4) of the Green's function, $N+m$ needs to be even.

Expressing the bulk-bulk and bulk-brane propagator as

$$
\begin{aligned}
G^{\chi\phi}(0) &= -i\lambda\tilde{D}G^{\chi\chi} \\
G^{\phi\phi}(0,0) &= \tilde{D} - \lambda^2\tilde{D}^2 G^{\chi\chi}\,,
\end{aligned}
$$

we get

$$
\begin{aligned}
G^{(N)} = &\sum_{n\geq 0}\sum_{m=0}^{n}\sum_{k=0}^{m}\sum_{\alpha=0}^{k/2}\sum_{\gamma=0}^{\alpha}\frac{(-i)^n}{n!}C_n^m C_m^k\beta_4^{n-m}\beta_3^{m-k}\lambda_3^k\lambda^{k-2\gamma}\tilde{D}^{k-2\gamma}\left(G^{\chi\chi}\right)^{\frac{1}{2}(N+4n-m-2\gamma)} \\
&\times \underbrace{(-1)^{\alpha-\gamma}(-i)^{k-2\alpha}C_k^{2\alpha}C_\alpha^\gamma\frac{(N-k+4n-m)!}{(N-2k+4n-m+2\alpha)!}P_{N-2k+4n-m+2\alpha}P_{2\alpha}}_{=F(\alpha)}.
\end{aligned}
\tag{B.6}
$$

Notice that the coefficient α does not affect the couplings, and so the summation over α can be performed without affecting the order of the diagram

$$
\sum_{\alpha=0}^{k/2}\sum_{\gamma=0}^{\alpha}F(\alpha) = \sum_{\gamma=0}^{k/2}\sum_{\alpha=\gamma}^{k/2}F(\alpha) = \sum_{\gamma=0}^{k/2}(-i)^{k-2\gamma}\frac{2^\gamma k!}{\gamma!(k-2\gamma)!}P_{N-m+4n+2\gamma}\,.
\tag{B.7}
$$

We now change the summation variables to $(n,m,k,\gamma)\to(X,Y,l,\gamma)$, with

$$
X = m-2\gamma\,,\quad Y = n-m+\gamma\,,\ \text{ and }\quad l = k-2\gamma
\tag{B.8}
$$

so that the Green's function can be expressed as

$$
\begin{aligned}
G^{(N)} = \sum_{X\geq 0}\sum_{Y\geq 0}\Bigg(&\frac{(-i)^{X+Y}}{X!Y!}P_{N+3X+4Y}\left(G^{\chi\chi}\right)^{\frac{1}{2}(N+3X+4Y)} \\
&\times\sum_{l=0}^{X}\sum_{\gamma=0}^{Y}C_X^l C_Y^\gamma\frac{(-i)^{l+\gamma}}{2^\gamma}\lambda^l\lambda_3^{l+2\gamma}\beta_3^{X-l}\beta_4^{Y-\gamma}\tilde{D}^{l+\gamma}\Bigg)\,,
\end{aligned}
\tag{B.9}
$$

Finally, summing over l and γ, we recover the familiar expressions

$$G^{(N)} = \sum_{X\geq 0}\sum_{Y\geq 0} \frac{(-i)^{X+Y}}{X!Y!} P_{N+3X+4Y} (G^{XX})^{\frac{1}{2}(N+3X+4Y)} \left(\beta_3 - i\lambda\lambda_3 \tilde{D}\right)^X \left(\beta_4 - \frac{i}{2}\lambda_3^2 \tilde{D}\right)^Y . \quad \text{(B.10)}$$

So the coupling constants λ, β_3, β_3, λ_3 and the free bulk-bulk propagator $\tilde{D}$ come in precisely the right combination to be finite. The RG flows of λ_3, β_3 and β_4 indeed ensures that both expressions $\left(\beta_3 - i\lambda\lambda_3\tilde{D}\right)$ and $\left(\beta_4 - \frac{i}{2}\lambda_3^2\tilde{D}\right)$ are finite, see eqs. (4.8) and (4.12). Since the renormalization of λ, m^2 and λ_2 is such that the brane propagator G^{XX} is finite, we can conclude that $G^{(N)}$ is completely finite in the thin brane limit (up to loop diagram divergences which can be renormalized in the standard way). This argument thus demonstrates that the theory is renormalizable.

References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[2] A. Salam and E. Sezgin, *Chiral compactification on Minkowski* $\times S^2$ *of* $N = 2$ *Einstein-Maxwell supergravity in six-dimensions*, *Phys. Lett.* **B 147** (1984) 47.

[3] P. Kanti, R. Madden and K.A. Olive, *A* 6*D brane world model*, *Phys. Rev.* **D 64** (2001) 044021 [hep-th/0104177];
G.W. Gibbons, R. Guven and C.N. Pope, 3*-branes and uniqueness of the Salam-Sezgin vacuum*, *Phys. Lett.* **B 595** (2004) 498 [hep-th/0307238];
Y. Aghababaie et al., *Warped brane worlds in six dimensional supergravity*, *JHEP* **09** (2003) 037 [hep-th/0308064];
H.M. Lee and A. Papazoglou, *Brane solutions of a spherical* σ*-model in six dimensions*, *Nucl. Phys.* **B 705** (2005) 152 [hep-th/0407208];
C.P. Burgess, F. Quevedo, G. Tasinato and I. Zavala, *General axisymmetric solutions and self-tuning in* 6*D chiral gauged supergravity*, *JHEP* **11** (2004) 069 [hep-th/0408109];
S. Mukohyama, Y. Sendouda, H. Yoshiguchi and S. Kinoshita, *Warped flux compactification and brane gravity*, *JCAP* **07** (2005) 013 [hep-th/0506050];
S.L. Parameswaran, G. Tasinato and I. Zavala, *The* 6*D superswirl*, *Nucl. Phys.* **B 737** (2006) 49 [hep-th/0509061];
H.M. Lee and C. Ludeling, *The general warped solution with conical branes in six-dimensional supergravity*, *JHEP* **01** (2006) 062 [hep-th/0510026];
A.J. Tolley, C.P. Burgess, D. Hoover and Y. Aghababaie, *Bulk singularities and the effective cosmological constant for higher co-dimension branes*, *JHEP* **03** (2006) 091 [hep-th/0512218];
A.J. Tolley, C.P. Burgess, C. de Rham and D. Hoover, *Scaling solutions to* 6*D gauged chiral supergravity*, *New J. Phys.* **8** (2006) 324 [hep-th/0608083];
T. Kobayashi and M. Minamitsuji, *Brane cosmological solutions in six-dimensional warped flux compactifications*, *JCAP* **07** (2007) 016 [arXiv:0705.3500];
E.J. Copeland and O. Seto, *Dynamical solutions of warped six dimensional supergravity*, *JHEP* **08** (2007) 001 [arXiv:0705.4169].

[4] S. Hayakawa, D. Ida, T. Shiromizu and T. Tanaka, *Gravitation in the codimension two brane world*, *Prog. Theor. Phys. Suppl.* **148** (2003) 128;
J.M. Cline, J. Descheneau, M. Giovannini and J. Vinet, *Cosmology of codimension-two braneworlds*, *JHEP* **06** (2003) 048 [hep-th/0304147];
J. Vinet, *Generalised cosmology of codimension-two braneworlds*, *Int. J. Mod. Phys.* **A 19** (2004) 5295 [hep-th/0408082];
C. Charmousis and R. Zegers, *Einstein gravity on an even codimension brane*, *Phys. Rev.* **D 72** (2005) 064005 [hep-th/0502171];
E. Papantonopoulos and A. Papazoglou, *Cosmological evolution of a purely conical codimension-2 brane world*, *JHEP* **09** (2005) 012 [hep-th/0507278].

[5] M.L. Graesser, J.E. Kile and P. Wang, *Gravitational perturbations of a six dimensional self-tuning model*, *Phys. Rev.* **D 70** (2004) 024008 [hep-th/0403074];
H.M. Lee and A. Papazoglou, *Scalar mode analysis of the warped Salam-Sezgin model*, *Nucl. Phys.* **B 747** (2006) 294 [*Erratum ibid.* **B 765** (2007) 200] [hep-th/0602208];
C.P. Burgess, C. de Rham, D. Hoover, D. Mason and A.J. Tolley, *Kicking the rugby ball: perturbations of 6D gauged chiral supergravity*, *JCAP* **02** (2007) 009 [hep-th/0610078];
S.L. Parameswaran, S. Randjbar-Daemi and A. Salvio, *Stability and negative tensions in 6D brane worlds*, arXiv:0706.1893.

[6] M. Peloso, L. Sorbo and G. Tasinato, *Standard 4D gravity on a brane in six dimensional flux compactifications*, *Phys. Rev.* **D 73** (2006) 104025 [hep-th/0603026].

[7] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315].

[8] A. Albrecht, C.P. Burgess, F. Ravndal and C. Skordis, *Natural quintessence and large extra dimensions*, *Phys. Rev.* **D 65** (2002) 123507 [astro-ph/0107573];
D.M. Ghilencea, D. Hoover, C.P. Burgess and F. Quevedo, *Casimir energies for 6D supergravities compactified on $\mathcal{T}_2/Z_N$ with Wilson lines*, *JHEP* **09** (2005) 050 [hep-th/0506164];
E. Elizalde, M. Minamitsuji and W. Naylor, *Casimir effect in rugby-ball type flux compactifications*, *Phys. Rev.* **D 75** (2007) 064032 [hep-th/0702098];
M. Minamitsuji, *Casimir effect in a 6D warped flux compactification model*, arXiv:0704.3623;
K. Koyama, *The cosmological constant and dark energy in braneworlds*, arXiv:0706.1557.

[9] V.A. Rubakov and M.E. Shaposhnikov, *Extra space-time dimensions: towards a solution to the cosmological constant problem*, *Phys. Lett.* **B 125** (1983) 139.

[10] Y. Aghababaie, C.P. Burgess, S.L. Parameswaran and F. Quevedo, *Towards a naturally small cosmological constant from branes in 6D supergravity*, *Nucl. Phys.* **B 680** (2004) 389 [hep-th/0304256];
O. Corradini, A. Iglesias and Z. Kakushadze, *Toward solving the cosmological constant problem?*, *Int. J. Mod. Phys.* **A 18** (2003) 3221 [hep-th/0212101];
S.M. Carroll and M.M. Guica, *Sidestepping the cosmological constant with football-shaped extra dimensions*, hep-th/0302067;
I. Navarro, *Codimension two compactifications and the cosmological constant problem*, *JCAP* **09** (2003) 004 [hep-th/0302129]; *Spheres, deficit angles and the cosmological constant*, *Class. and Quant. Grav.* **20** (2003) 3603 [hep-th/0305014];
H.-P. Nilles, A. Papazoglou and G. Tasinato, *Selftuning and its footprints*, *Nucl. Phys.* **B 677** (2004) 405 [hep-th/0309042];

C.P. Burgess, *Supersymmetric large extra dimensions and the cosmological constant: an update*, *Ann. Phys. (NY)* **313** (2004) 283 [hep-th/0402200];
J.-M. Schwindt and C. Wetterich, *The cosmological constant problem in codimension-two brane models*, *Phys. Lett.* **B 628** (2005) 189 [hep-th/0508065];
C.P. Burgess, *Supersymmetric large extra dimensions and the cosmological constant problem*, hep-th/0510123;
E.K. Park and P.S. Kwon, *A self-tuning mechanism in* 6*D gravity-scalar theory*, *JHEP* **11** (2007) 051 [hep-th/0702171].

[11] C.P. Burgess, J. Matias and F. Quevedo, *MSLED: a minimal supersymmetric large extra dimensions scenario*, *Nucl. Phys.* **B 706** (2005) 71 [hep-ph/0404135];
J. Vinet and J.M. Cline, *Can codimension-two branes solve the cosmological constant problem?*, *Phys. Rev.* **D 70** (2004) 083514 [hep-th/0406141];
J. Garriga and M. Porrati, *Football shaped extra dimensions and the absence of self-tuning*, *JHEP* **08** (2004) 028 [hep-th/0406158];
J. Vinet and J.M. Cline, *Codimension-two branes in six-dimensional supergravity and the cosmological constant problem*, *Phys. Rev.* **D 71** (2005) 064011 [hep-th/0501098].

[12] G. Dvali, G. Gabadadze and M. Shifman, *Diluting cosmological constant in infinite volume extra dimensions*, *Phys. Rev.* **D 67** (2003) 044020 [hep-th/0202174];
O. Corradini, A. Iglesias and Z. Kakushadze, *Diluting solutions of the cosmological constant problem*, *Mod. Phys. Lett.* **A 18** (2003) 1343 [hep-th/0305164].

[13] G. Dvali, S. Hofmann and J. Khoury, *Degravitation of the cosmological constant and graviton width*, *Phys. Rev.* **D 76** (2007) 084006 [hep-th/0703027].

[14] G.R. Dvali, G. Gabadadze and M. Porrati, *Metastable gravitons and infinite volume extra dimensions*, *Phys. Lett.* **B 484** (2000) 112 [hep-th/0002190]; 4*D gravity on a brane in* 5*D Minkowski space*, *Phys. Lett.* **B 485** (2000) 208 [hep-th/0005016];
G.R. Dvali and G. Gabadadze, *Gravity on a brane in infinite-volume extra space*, *Phys. Rev.* **D 63** (2001) 065007 [hep-th/0008054].

[15] S. Weinberg, *The cosmological constant problem*, *Rev. Mod. Phys.* **61** (1989) 1.

[16] R. Geroch and J.H. Traschen, *Strings and other distributional sources in general relativity*, *Phys. Rev.* **D 36** (1987) 1017.

[17] W.D. Goldberger and M.B. Wise, *Renormalization group flows for brane couplings*, *Phys. Rev.* **D 65** (2002) 025011 [hep-th/0104170].

[18] W.D. Goldberger and I.Z. Rothstein, *An effective field theory of gravity for extended objects*, *Phys. Rev.* **D 73** (2006) 104029 [hep-th/0409156]; *Dissipative effects in the worldline approach to black hole dynamics*, *Phys. Rev.* **D 73** (2006) 104030 [hep-th/0511133]; *Towers of gravitational theories*, *Gen. Rel. Grav.* **38** (2006) 1537 [*Int. J. Mod. Phys.* **D 15** (2006) 2293] [hep-th/0605238];
W.D. Goldberger, *Les Houches lectures on effective field theories and gravitational radiation*, hep-ph/0701129.

[19] R.A. Porto, *Post-newtonian corrections to the motion of spinning bodies in NRGR*, *Phys. Rev.* **D 73** (2006) 104031 [gr-qc/0511061];
R.A. Porto and I.Z. Rothstein, *The hyperfine Einstein-Infeld-Hoffmann potential*, *Phys. Rev. Lett.* **97** (2006) 021101 [gr-qc/0604099];

R.A. Porto and R. Sturani, *Scalar gravity: post-newtonian corrections via an effective field theory approach*, `gr-qc/0701105`; *Scalar gravity: post-newtonian corrections via an effective field theory approach*, `gr-qc/0701105`.

[20] F. del Aguila, M. Perez-Victoria and J. Santiago, *Bulk fields with general brane kinetic terms*, *JHEP* **02** (2003) 051 [`hep-th/0302023`]; *Effective description of brane terms in extra dimensions*, *JHEP* **10** (2006) 056 [`hep-ph/0601222`].

[21] O. Corradini, A. Iglesias, Z. Kakushadze and P. Langfelder, *A remark on smoothing out higher codimension branes*, *Mod. Phys. Lett.* **A 17** (2002) 795 [`hep-th/0201201`].

[22] T. Gherghetta and M.E. Shaposhnikov, *Localizing gravity on a string-like defect in six dimensions*, *Phys. Rev. Lett.* **85** (2000) 240 [`hep-th/0004014`].

[23] I. Navarro and J. Santiago, *Gravity on codimension* 2 *brane worlds*, *JHEP* **02** (2005) 007 [`hep-th/0411250`];
C. de Rham and A.J. Tolley, *Gravitational waves in a codimension two braneworld*, *JCAP* **02** (2006) 003 [`hep-th/0511138`].

[24] T. Kobayashi and M. Minamitsuji, *Gravity on an extended brane in six-dimensional warped flux compactifications*, *Phys. Rev.* **D 75** (2007) 104013 [`hep-th/0703029`];
C.P. Burgess, D. Hoover and G. Tasinato, *UV caps and modulus stabilization for* 6*D gauged chiral supergravity*, *JHEP* **09** (2007) 124 [`arXiv:0705.3212`].

[25] A.E. Nelson, *A new angle on intersecting branes in infinite extra dimensions*, *Phys. Rev.* **D 63** (2001) 087503 [`hep-th/9909001`];
E. Gravanis and S. Willison, *Intersecting hyper-surfaces in dimensionally continued topological density gravitation*, *J. Math. Phys.* **45** (2004) 4223 [`hep-th/0306220`];
I. Navarro and J. Santiago, *Higher codimension braneworlds from intersecting branes*, *JHEP* **04** (2004) 062 [`hep-th/0402204`];
N. Kaloper, *Origami world*, *JHEP* **05** (2004) 061 [`hep-th/0403208`].

[26] C. de Rham et al., *Cascading DGP*, `arXiv:0711.2072`;
C. de Rham, S. Hofmann, J. Khoury and A.J. Tolley, *Cascading gravity and degravitation*, `arXiv:0712.2821`.

[27] S. Weinberg, *Infrared photons and gravitons*, *Phys. Rev.* **140** (1965) B516.

[28] C. de Rham, T. Shiromizu and A.J. Tolley, *Weaker gravity at submillimetre scales in braneworlds models*, `gr-qc/0604071`.

[29] C. Deffayet, G.R. Dvali and G. Gabadadze, *Accelerated universe from gravity leaking to extra dimensions*, *Phys. Rev.* **D 65** (2002) 044023 [`astro-ph/0105068`].

[30] C. de Rham and A.J. Tolley, *Mimicking lambda with a spin-two ghost condensate*, *JCAP* **07** (2006) 004 [`hep-th/0605122`];
G. Gabadadze, *A model for cosmic self-acceleration*, `hep-th/0612213`.

[31] E.J. Copeland, M. Sami and S. Tsujikawa, *Dynamics of dark energy*, *Int. J. Mod. Phys.* **D 15** (2006) 1753 [`hep-th/0603057`].

[32] P.H. Beauchemin, G. Azuelos and C.P. Burgess, *Dimensionless coupling of bulk scalars at the LHC*, *J. Phys.* **G 30** (2004) N17 [`hep-ph/0407196`];
J.J. van der Bij and S. Dilcher, *A higher dimensional explanation of the excess of Higgs-like events at CERN LEP*, *Phys. Lett.* **B 638** (2006) 234 [`hep-ph/0605008`].